Analytic models for orogenic collapse

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Abstract

Orogenic plateaus and collisional mountain belts are formed by tectonic driving forces, such as those associated with a continental collision, and magmatism. When the horizontal driving forces are removed, e.g., at the conclusion of an orogeny, the elevation of the plateau or collisional orogen will relax by one or both of two processes. The first of these is erosion which removes the near surface rocks and allows uplift. The second is the gravitational collapse of elevated topography. In this paper we consider a simplified analysis in order to quantify the relative importance of erosion and gravitational collapse in the removal of topography. We treat the erosion problem using the Culling (diffusion equation) approach. The basic assumption is that the material transport down slope is proportional to the slope with the equivalent coefficient of diffusion, $K$, the constant of proportionality. The parameter $K$ quantifies the rate of erosion. It is applicable whether or not the topography is compensated. For the gravitational collapse problem, we use the thin viscous sheet approximation and assume a linear viscous rheology. To simplify the analysis, we assume the thickness of the continental crust approximates the thickness of the highly viscous lithosphere, and that the crust overlies a much lower viscosity mantle. We also assume the topography is harmonic with a wavelength, $\lambda$. The viscosity, $\eta$ quantifies the thinning of the crust due to the buoyancy driven lateral flow. Our results indicate an exponential decay of topography with time and show that the relative role of erosion versus gravitational collapse for compensated topography is controlled by the value of the non-dimensional collapse number $C = gh_0 \lambda^2 \rho_c/16K\eta \pi^2$, where $g$ is the acceleration due to gravity, $h_0$ is a reference thickness of the crust, $\rho_c$ is the density of the crust, and $\lambda$, $K$, and $\eta$ are as defined above. If $C$ is large compared with unity gravitational collapse dominates, if $C$ is small compared with unity erosion dominates.

Keywords: Orogenic collapse; Gravitational collapse; Tibetan Plateau; Erosion; Analytic model

1. Introduction

Orogenic plateaus are broad regions of elevated topography attributed to compressional tectonic processes and volcanism. An example is the Tibetan Plateau, which has dimensions of about 1500 km by 3500 km and average elevations on the order of 5 km (Fielding et al., 1994). The formation of the Tibetan Plateau is a result of the collision between the Indian and Eurasian plates that initiated 50–70 Myr ago (Yin and
When the continental collision between the Indian and Eurasian plates terminates, the height of topography of Tibet will disappear over a period of time. An example of this process in the past is the surface relaxation of the Appalachian highlands in the early to mid-Mesozoic after the end of the last continental collision between the Eurasian and North American plates (Dewey, 1988). The purpose of this paper is to quantify the relative roles of erosion and gravitational collapse in the removal of this type of topography. We will first examine erosion, then gravitational collapse, and finally, we consider these two processes simultaneously.

To state that erosion is a complex process is clearly an understatement. Both chemical and mechanical processes are operative. In addition, mechanisms such as landsliding and glaciation play important roles in the evolution of a landscape. Burbank (2002) provides a comprehensive review of erosional processes. Of the variety of theoretical approaches to erosion, one of the most widely used is the Culling model. In this model (Culling, 1960, 1963) the rate of material transport is assumed to be proportional to the slope. With this assumption, erosion satisfies the heat (diffusive transfer) equation and is controlled by the magnitude of the transport coefficient, $K$. From studies of foreland basins Flemings and Jordan (1989) conclude that $K$ is in the range of $10^2$–$10^3$ m$^2$/yr, and from studies of erosion as a driving mechanism for mountain growth Avouac and Burov (1996) conclude that $K$ is in the range of $10^3$–$10^4$ m$^2$/yr.

Although, the Culling approach can be criticized as being simplistic, there is really no alternative approach available. Solutions of the heat equation have been successful in explaining the morphology of alluvial fans, prograding river deltas, and eroding fault scarps (Wallace, 1977; Nash, 1980a,b; Begin et al., 1981; Gill, 1983a,b; Hanks et al., 1984; Kenyon and Turcotte, 1985; Hanks and Wallace, 1985; Hanks and Andrews, 1989). In addition, a uniform feature of topography along a linear track is that it is well approximated by a Brownian walk time series. This is true for scales from meters to the radius of the Earth (Turcotte, 1997). Pelletier and Turcotte (1996, 1997) showed that this behavior can be obtained using the Culling diffusion model with a white noise random driver. In the physics literature this is known as the Edwards–Wilkinson equation and it is widely used to quantify stochastic surface growth (Barabasi and Stanley, 1993). Based on the current state of knowledge we conclude that it is appropriate to use the Culling quantification of erosion in our analysis.

An alternative mechanism for the removal of topography is gravitational collapse. This mechanism can lead to the total loss of topography with no erosion. Gravitational collapse is a very complex problem involving both brittle and ductile deformation and has been studied by a number of workers (i.e, Dalmayrac and Molnar, 1981; Molnar and Lyon-Caen, 1988; Dewey, 1988; Gaudemer et al., 1988). One approach to model gravitational collapse is to treat the lithosphere as a highly viscous material in which a lateral gradient in gravitational potential energy, due to the elevated topography, provides a driving mechanism for the lateral flow of the lithosphere from high pressure (high elevations) to low pressure (lower elevations) (Frank, 1972; England and McKenzie, 1982; Fleitout and Froidevaux, 1982, 1983; England, 1987; Fleitout, 1991; Rey et al., 2001; Turcotte and Schubert, 2002), although, models for the buoyancy driven flows may vary in the treatment of the number of rheologic layers into which the lithosphere is divided as well as the rheology of these layers (Bird, 1991). In addition, Zhou and Sandiford (1992) have carried out a detailed study of buoyancy driven flows using a stress envelope for the lithospheric rheology that includes friction constraints at shallow depths and thermally activated creep at greater depths. They found that the collapse rate is sensitive to the Moho temperature.

Clearly, the deformation of the brittle upper lithosphere is dominated by fracture and faulting, however, this deformation takes place on a wide range of scales, and therefore it is often appropriate to model it as a continuum deformation. This problem has been recently treated by Nanjo and Turcotte (2005) and by Nanjo et al. (2005) using concepts of damage mechanics. These authors show that it is appropriate to treat continuum brittle deformation using a nonlinear viscous rheology with a yield stress.

Despite the wide range of complex models available to study buoyancy driven flows in the crust and mantle, the thin viscous sheet models have been quite successfully applied to problems of lithospheric deformation (England and McKenzie, 1982; England and Houseman, 1986; Houseman and England, 1986; Sonder and England, 1986). In the thin viscous sheet model, the deforming lithosphere is assigned a uniform viscous rheology. The lubrication approximation is utilized so that only gravitational and viscous forces are considered. Also vertical velocities are assumed to be small compared with horizontal velocities. Using estimated strain rates for Tibet, England and Molnar (1997) conclude that for strain rates of $10^{-15}$–$10^{-16}$ s$^{-1}$ an effective viscosity of $5 \times 10^{21}$–$2 \times 10^{22}$ Pa s is
appropriate for the deformation of the Tibetan lithosphere. More recently, Flesch et al. (2001) have considered the dynamics of the India–Asia collision zone and concluded that a vertically averaged viscosity in the range of $0.5 \times 10^{22} – 5 \times 10^{22}$ Pa s is appropriate for Tibet.

Several authors have previously considered the relative roles of both erosion and continuum deformation in tectonics. Similar to the approach we take in this paper for the erosion problem, Avouac and Burov (1996) and Jimenez-Munt et al. (2005) treated erosion using the Culling theory. However, these authors used complex tectonic models to quantify the continuum deformation. Our objective in this paper is to present an analytical model that provides a quantitative measure of the relative roles of erosion and gravitational collapse in the loss of topography. We assume that: (1) the topography is isostatically compensated, (2) we apply the Culling model to quantify erosion (exhumation), (3) we use a Newtonian viscous rheology and apply the thin viscous sheet model to quantify gravitational collapse, and (4) we combine the Culling model with the thin viscous sheet approximation to quantify the net surface relaxation, i.e., the combined erosion and gravitational collapse.

Before presenting our analysis we must clarify the relative roles of erosion and gravitational collapse. We follow the approach of England and Molnar (1990) to uplift and erosion, in which surface uplift = uplift of rock – exhumation. Since our analysis describes the decay of topography, we use the term “surface relaxation” in place of “surface uplift” to describe the change in height of the surface with respect to the geoid. In our paper we define the surface relaxation velocity, $v_r$, as the rate at which topography is lost. We define the erosion rate, $v_e$, as the velocity at which rock is removed from the surface, i.e., the rate of exhumation. For the case of uncompensated topography with erosional collapse only, the uplift of rock equals zero, and the surface relaxation velocity equals the erosion rate, i.e., $v_r = v_e$, whereas, for compensated topography with erosional collapse only, the erosion rate, $v_e$, will be $3$ to $6$ times greater than the surface relaxation velocity, $v_r$, due to the uplift of the lower crust. Alternatively, gravitational collapse can lead to the total loss of topography with no erosion. In our calculations for gravitational collapse only, we define the gravitational collapse rate, $v_g$, as the velocity at which topography is lost relative to the geoid due to gravitational processes, i.e., the rate of surface relaxation in which the rate of exhumation equals zero. In our analysis of combined erosion and gravitational collapse, the surface relaxation velocity, $v_r$, is a function of both erosion and gravitational collapse.

2. Model description

Our two-dimensional model contains two layers, the crust and underlying mantle with densities $\rho_c$ and $\rho_m$, respectively, as illustrated in Fig. 1. We assume the thickness of the crust is equivalent to that of the highly viscous lithosphere. In general the thickness of the flowing lithosphere may be different than the thickness of the crust. However, to simplify our analysis we assume that gravitationally induced flows are restricted to the continental crust. The crust is assigned a linear viscosity, $\eta$, which is large compared with the equivalent viscosity of the underlying mantle. To further simplify the analysis, we assume that gravitational collapse occurs when the horizontal driving forces are removed, e.g., at the termination of an orogeny.

We assume that the crustal thickness, $h(x,t)$, is harmonic and is given by

$$h(x,t) = [h(x=0,t) - h_o] \cos\left(\frac{2\pi X}{\lambda}\right) + h_o$$

where $h_o$ is the thickness of the crust with zero elevation and no crustal root, and $\lambda$ is the wavelength. The height of topography, $w(x,t)$, also satisfies the harmonic relation

$$w(x,t) = w(x=0,t) \cos\left(\frac{2\pi X}{\lambda}\right).$$

Assuming Airy isostasy, the thickness of the crust is related to the height of topography by

$$w(x,t) = [h(x,t) - h_o] \left(1 - \frac{\rho_c}{\rho_m}\right).$$

Since our equations are linear, solutions for different wavelengths can be superimposed without loss of generality for an arbitrary original topography.

Fig. 1. Illustration of the model. The model contains two layers, the isostatically compensated crust and the underlying mantle of densities, $\rho_c$ and $\rho_m$, respectively. We assume the thickness of the crust is equivalent to the thickness of the highly viscous lithosphere. In addition, $h$, $h_o$, $w$, and $\lambda$ are the thickness of the compensated crust, the thickness of the reference crust, the height of the harmonic topography, and the wavelength of the plateau, respectively.
A different approach would be to consider an expanding region into which the flow associated with the buoyancy driving force would spread. However, this would require a numerical solution. Our periodic formulation is approximate but should well represent the horizontal flows associated with elevated topography as well as a sink for the horizontal transport of sediments.

3. Erosional collapse

We first consider the case in which the reduction in elevation (loss of topography) is due to erosion only. We follow Avouac and Burov (1996) and Jimenez-Munt et al. (2005) in applying the Culling (1960, 1963) theory for erosion. The main assumption in this approach is that the down-slope flux of eroded material is linearly proportional to the topographic slope (see also Turcotte and Schubert, 2002). The rate of erosion, \( \frac{dh(x,t)}{dt} \), is given by

\[
\frac{dh(x,t)}{dt} = K \frac{\partial^2 w(x,t)}{\partial x^2} \tag{4}
\]

where \( K \) is the transport coefficient. For uncompensated topography, \( w(x,t)=h(x,t)-h_o \), and Eq. (4) reduces to the heat equation. However, for compensated topography \( w(x,t) \) is defined as in Eq. (3), and the rate of erosion is proportional to the second spatial derivative of the height of the topography.

Substitution of Eq. (1) into Eq. (3) and then the result into Eq. (4) gives the differential equation for the rate of erosion at \( x=0 \),

\[
\frac{dh(x=0,t)}{dt} = -\frac{4\pi^2}{\lambda^2} K \left(1-\frac{\rho_c}{\rho_m}\right) [h(x=0,t)-h_o]. \tag{5}
\]

Integration of Eq. (5), with the initial condition \( h_1=h(x=0,t=0) \) gives the solution

\[
h(x=0,t) = (h_1-h_o) \exp \left(-\frac{t}{\tau_e}\right) + h_o \tag{6}
\]

where the characteristic time for erosion, \( \tau_e \), is given by

\[
\tau_e = \frac{\lambda^2}{4\pi^2 K \left(1-\frac{\rho_c}{\rho_m}\right)}. \tag{7}
\]

Substituting Eq. (6) into Eq. (1) gives an expression for the crustal thickness as a function of position and time

\[
h(x,t) = (h_1-h_o) \exp \left(-\frac{t}{\tau_e}\right) \cos \left(\frac{2\pi x}{\lambda}\right) + h_o. \tag{8}
\]

Substitution of this expression into Eq. (3) gives the following expression for the topography as a function of position and time

\[
w(x,t) = w_1 \exp \left(-\frac{t}{\tau_e}\right) \cos \left(\frac{2\pi x}{\lambda}\right) \tag{9}
\]

where we define \( w_1=w(x=0,t=0)=(h_1-h_o)(1-\rho_c/\rho_m) \). These results give an exponential decay of topography with time that depends on the characteristic time for erosion, \( \tau_e \).

The maximum surface relaxation velocity, \( v_{\text{max}} \), at \( x=0 \) and \( t=0 \), is obtained from Eq. (9) with the result

\[
v_{\text{max}} = -\frac{\partial w}{\partial t}|_{x=0,t=0} = \frac{w_1}{\tau_e}. \tag{10}
\]

The maximum erosion rate, \( v_{\text{max}} \), at \( x=0 \) and \( t=0 \), is obtained from Eq. (8) with the result

\[
v_{\text{max}} = v_{\text{max}} \left(1-\frac{\rho_c}{\rho_m}\right)^{-1} \tag{11}
\]

as expected.

As a specific example, we take \( \lambda=2000 \) km, \( \rho_c=2800 \) kg/m³, and \( \rho_m=3300 \) kg/m³ and give the dependence of the characteristic erosion time from Eq. (7) on the erosion coefficient in Fig. 2. The characteristic time for erosion is seen to decrease with increasing erosion coefficient. We next calculate the dependence of the maximum erosion rate, \( v_{\text{max}} \), on the erosion coefficient from Eq. (11) for the parameter

![Fig. 2. Dependence of the characteristic time for erosion, \( \tau_e \), on the erosion coefficient, \( K \), for the case of erosion only (no gravitational collapse).](Image)
values given above and for a range in the initial maximum height of topography, \( w_1 \), and show the result in Fig. 3. For a given erosion coefficient, the greater the initial maximum height of topography, the greater the maximum erosion rate. Avouac and Burov (1996) and Burbank (2002) have given comprehensive reviews of erosion rates and a reasonable range of values appears to be \( 0.1 < v_{\text{emax}} < 10 \, \text{mm/yr} \). From Fig. 3 we see that for the corresponding range in erosion rate, the transport coefficient is on the order of \( 2 \times 10^3 < K < 5 \times 10^5 \, \text{m}^2/\text{yr} \).

We now choose specific values for the characteristic time for erosion, \( \tau_e \), and calculate the corresponding values of the erosion coefficient, maximum surface relaxation velocity, and maximum erosion rate. We assume the maximum height of topography, \( w_1 \), is equal to 5 km, a value that is appropriate for the Tibetan plateau (Fielding et al., 1994). Note that assuming Airy isostasy, a topographic amplitude of 5 km and a reference thickness, \( h_o \), of 40 km correspond to a crustal thickness of 73 km, for the densities assumed above. This thickness is within the range of the crustal thicknesses obtained for Tibet from seismic data (Yuan et al., 1997; Zhang and Klemperer, 2005). Taking \( \tau_e = 10^7 \, \text{yr} \) and using the parameter values above in Eq. (7), the resulting value for \( K \) is \( 6.7 \times 10^3 \, \text{m}^2/\text{yr} \). We find from Eq. (10) that the maximum surface relaxation velocity is \( v_{\text{max}} = 0.5 \, \text{mm/yr} \) and from Eq. (12) that the maximum erosion rate is \( v_{\text{emax}} = 3.3 \, \text{mm/yr} \). Now taking \( \tau_e = 10^8 \, \text{yr} \), we find from Eq. (7) that \( K = 6.7 \times 10^3 \, \text{m}^2/\text{yr} \). Again, assuming the initial maximum height of topography, \( w_1 = 5 \, \text{km} \), we find from Eq. (10) that the surface relaxation velocity is \( v_{\text{max}} = 0.05 \, \text{mm/yr} \) and from Eq. (12) that the maximum erosion rate is \( v_{\text{emax}} = 0.33 \, \text{mm/yr} \). Thus, the larger the characteristic erosion time, the smaller the maximum erosion rate and the smaller the surface relaxation velocity. In addition the maximum erosion rates calculated are approximately 6.5 times greater than the maximum surface relaxation velocities because of the isostatic uplift of the crust.

4. Gravitational collapse

We next consider the case in which the loss of topography is due to the gravitational collapse of the continental crust. We assume a uniform linear viscous rheology for the crust and that the crust overlies a weak (much lower viscosity) mantle. We apply the thin viscous sheet approach and assume the lubrication approximation is valid (England and McKenzie, 1982; Houseman and England, 1986; Sonder and England, 1986). In the lubrication approximation inertial forces are neglected and the vertical component of velocity, \( v \), is negligible compared with the horizontal component, \( u \). The conservation of mass requires

\[
\frac{\partial h(x,t)}{\partial t} + \frac{\partial}{\partial x} [u(x,t)h(x,t)] = 0 \tag{13}
\]

where \( h(x,t) \) is the thickness of isostatically compensated crust, and the horizontal velocity, \( u(x,t) \) is assumed to be independent of \( y \). The conservation of the force balance in the \( x \)-direction requires

\[
\frac{\partial}{\partial x} \left[ 4\eta h(x,t) \frac{\partial u(x,t)}{\partial x} - \rho_c \left( 1 - \frac{\rho_c}{\rho_m} \right) g h(x,t)^2 \right] = 0 \tag{14}
\]

where \( \eta \) is the crustal viscosity, and \( g \) is the acceleration due to gravity. The first term in Eq. (14) is the viscous resistance to flow, and the second term is the gravitational buoyancy force that will lead to the collapse of elevated topography.

Both Eqs. (13) and (14) are nonlinear and we linearize them assuming shallow topography. Therefore, consistent with the lubrication approximation and the neglect of the vertical velocity component we linearize Eqs. (13) and (14) by introducing

\[
h(x,t) = h_o + h'(x,t) \tag{15}
\]

\[
u(x,t) = u'(x,t) \tag{16}
\]

where \( h'(x,t) \) is the variation in crustal thickness which includes both the topographic deflection and the crustal root, and \( h'(x,t)/h_o \ll 1 \). We recognize that the crustal thicknesses in orogenic plateaus may be double the reference thickness, so that the linearization may be a poor approximation. However, our primary objective is to quantify the relative importance of the erosional and
gravitational collapse mechanisms, and the linearization approximation allows us to obtain analytic rather than numerical results. Thus, substituting Eqs. (15) and (16) into Eqs. (13) and (14) and neglecting quadratic terms, the conservation equations become

\[
\frac{\partial h'(x,t)}{\partial t} + h_o \frac{\partial u'(x,t)}{\partial x} = 0
\]  

(17)

and

\[
4\eta h_o \frac{\partial^2 u'(x,t)}{\partial x^2} - \rho_c \left(1 - \frac{\rho_c}{\rho_m}\right) g h_o \frac{\partial h'(x,t)}{\partial x} = 0.
\]  

(18)

We assume that the linearized variation in crustal thickness, \(h'(x,t)\), and horizontal velocity, \(u'(x,t)\), are harmonic and write

\[
h'(x,t) = h'(0,t)\cos\left(\frac{2\pi x}{\lambda}\right)
\]  

(19)

\[
u'(x,t) = u'(0,t)\sin\left(\frac{2\pi x}{\lambda}\right)
\]  

(20)

where \(h'(x,t) = h(x=0,t) - h_o\). Note that the velocity is a maximum where the change in crustal thickness is a maximum as illustrated in Fig. 4. This occurs at \(x = (1+2n)\lambda/4\), where \(n\) is an integer. Taking the appropriate derivatives of Eqs. (19) and (20) and substituting the results into the linearized conservation of mass equation, Eq. (17), gives

\[
\frac{dh'(x=0,t)}{dt} + \frac{2\pi h_o}{\lambda} u'(x=0,t) = 0.
\]  

(21)

Substitution of the appropriate derivatives of Eqs. (19) and (20) into the linearized force balance equation, Eq. (18), and rearranging terms gives an expression for the linearized horizontal velocity at \(x=0\)

\[
u'(x=0,t) = \frac{\rho_c g \left(1 - \frac{\rho_c}{\rho_m}\right)}{8\pi\eta} h'(x = 0,t).
\]  

(22)

Combining Eqs. (21) and (22) yields a differential equation that can be solved for the linearized variation in crustal thickness, \(h'(x=0,t)\), at \(x=0\),

\[
\frac{dh'(x=0,t)}{dt} + \frac{\rho_c h_o g}{4\eta} \left(1 - \frac{\rho_c}{\rho_m}\right) h'(x = 0,t) = 0.
\]  

(23)

Integration of Eq. (23) with the initial condition \(h'(x=0,t=0) = h_1 - h_o\), where \(h_1 = h(x=0,t=0)\) yields the following solution for the linearized variation in crustal thickness,

\[
h'(x=0,t) = (h_1 - h_o)\exp\left(-\frac{t}{\tau_g}\right)
\]  

(24)

where the characteristic time for gravitational collapse, \(\tau_g\), is given by

\[
\tau_g = \frac{4\eta}{\rho_c h_o g \left(1 - \frac{\rho_c}{\rho_m}\right)}.
\]  

(25)

The form of Eq. (25) indicates that the characteristic time for gravitational collapse is linearly proportional to the viscosity. Thus, an increase in the crustal viscosity will increase the length of time it takes for a plateau to gravitationally collapse, as expected.

We obtain a solution for the total crustal thickness, \(h(x,t)\), by the substitution of Eq. (24) into Eq. (19) and then the substitution of this result into Eq. (15)

\[
h(x,t) = (h_1 - h_o)\exp\left(-\frac{t}{\tau_g}\right) \cos\left(\frac{2\pi x}{\lambda}\right) + h_o.
\]  

(26)

Substitution of Eq. (26) into Eq. (3) yields the solution for the height of topography, \(w(x,t)\),

\[
w(x,t) = w_1 \exp\left(-\frac{t}{\tau_g}\right) \cos\left(\frac{2\pi x}{\lambda}\right)
\]  

(27)

where \(w_1 = w(x=0,t=0) = (h_1 - h_o)(1 - \rho_c/\rho_m)\). Eq. (27) indicates an exponential decay of the height of topography with time. Substitution of Eq. (24) into Eq. (22) and the result into Eq. (20), and using the
Fig. 5. Dependence of the characteristic time for gravitational collapse, \( \tau_g \), on the viscosity of the crust, \( \eta \), for the case of gravitational collapse only (no erosion).

definition in Eq. (16), yields the following solution for the horizontal velocity,

\[
u(x,t) = h_1 - h_o \frac{\rho_c g (1-\rho_c/\rho_m) \lambda}{8\pi \eta} \exp \left( -\frac{t}{\tau_g} \right) \sin \left( \frac{2\pi x}{\lambda} \right).
\] (28)

Eq. (28) demonstrates the inverse dependence of the horizontal velocity on the viscosity. Increasing the viscosity, i.e., increasing the internal resistance to flow, decreases the rate of the buoyancy driven lateral flow of the crust. The maximum velocity of the removal of topography due to gravitational collapse only, \( v_{g\text{max}} \), at \( x=0 \) and \( t=0 \), is obtained from Eq. (27) with the result

\[
\nu_{\text{max}} = v_{g\text{max}} = -\frac{\partial w}{\partial t} \bigg|_{x=0,t=0} = \frac{w_1}{\tau_g},
\] (29)

where \( \nu_{\text{max}} = v_{g\text{max}} \), because there is no erosion.

With buoyancy driven crustal flows, although there is crustal thinning and loss of elevation, there is no erosion. We now calculate the dependence of the characteristic time for gravitational collapse, \( \tau_g \), on the crustal viscosity, \( \eta \) from Eq. (25), assuming the same values for \( \lambda \), \( h_o \), \( \rho_c \), and \( \rho_m \) as in the erosion example, and show the result in Fig. 5. For a viscosity range on the order of \( 1 \times 10^{19} \) to \( 1 \times 10^{24} \) Pa s, the corresponding range in the characteristic time for gravitational collapse is \( 1 \times 10^4 \) to \( 1 \times 10^9 \) yr. The larger the viscosity, the greater the characteristic time for gravitational collapse, and the smaller the buoyancy driven reduction in topography.

We now specify the characteristic time for gravitational collapse and determine the corresponding values of viscosity and surface relaxation velocity for a plateau of comparable scale to the Tibetan plateau. We first take \( \tau_g = 10^7 \) yr and find from Eq. (25) that \( \eta = 1.3 \times 10^{22} \) Pa s. Assuming the initial maximum height of topography of 5 km, we find from Eq. (29) that the maximum surface relaxation velocity is \( v_{r\text{max}}=v_{g\text{max}}=0.5 \) mm/yr. Taking \( \tau_g = 10^8 \) yr, we find from Eq. (25) that \( \eta = 1.3 \times 10^{23} \) Pa s and from Eq. (29) that \( v_{r\text{max}}=v_{g\text{max}}=0.05 \) mm/yr. The viscosity corresponding to the characteristic time for gravitational collapse of \( 10^7 \) yr is similar to viscosities estimated for the deformation of Tibet (England and Molnar, 1997; Flesch et al., 2001), and, thus, buoyancy driven flows appear to be a viable mechanism for the collapse of topography.

5. Combined erosion and gravitational collapse

To investigate the relative importance of erosion and gravitational collapse in the removal of topography, we now derive an analytical solution that is a function of both erosion and buoyancy driven flow. Since the governing equations for both erosion and gravitational collapse as given above are linear, they can be combined in a straightforward manner. Combining the conservation of mass equations for the two cases, Eqs. (4) and (17), and using the definitions in Eqs. (3) and (15) we obtain

\[
\frac{\partial h'(x,t)}{\partial t} + h_o \frac{\partial u'(x,t)}{\partial x} = K \frac{\partial^2 w(x,t)}{\partial x^2} = \left( 1 - \frac{\rho_c}{\rho_m} \right) K \frac{\partial^2 h'(x,t)}{\partial x^2}.
\] (30)

Thus, the sum of the variation of the crustal thickness with time and the horizontal flux of material is proportional to the second spatial derivative of the isostatically compensated topography.

Again we assume the harmonic forms of the variations in crustal thickness, \( h'(x,t) \), and horizontal velocity, \( u'(x,t) \). Taking the appropriate derivatives of Eqs. (19) and (20) and substituting the results into the combined conservation of mass equation, Eq. (30), gives

\[
\frac{dh'(x=0,t)}{dt} + \frac{2\pi}{\lambda} h_o u'(x=0,t) = -K \left( 1 - \frac{\rho_c}{\rho_m} \right) \frac{4\pi^2}{\lambda^2} h'(x=0,t).
\] (31)

The force balance given in Eq. (18) for gravitational collapse remains valid, and the expression for the linearized horizontal velocity given in Eq. (22) remains unchanged.
Thus, the substitution of Eq. (22) into Eq. (31) yields the following differential equation that can be solved for the linearized variation in crustal thickness $h'(x=0,t)$,

$$\frac{dh'(x=0,t)}{dt} = -\left[K\left(1-\frac{\rho_c}{\rho_m}\right)\frac{4\pi^2}{\lambda^2}h'(x=0,t) + \frac{\rho_cgh_o}{4\eta}\left(1-\frac{\rho_c}{\rho_m}\right)\right].$$  \hspace{1cm} (32)

Integration of Eq. (32) with the initial condition $h'(x=0,t=0)=(h_1-h_o)$ yields

$$h'(x=0,t) = (h_1-h_o)\exp\left(-\frac{t}{\tau_{eg}}\right)$$  \hspace{1cm} (33)

where the characteristic time for combined erosional and gravitational collapse, $\tau_{eg}$, is given by

$$\tau_{eg} = \frac{\lambda^2}{4\pi^2K\left(1-\frac{\rho_c}{\rho_m}\right)(1+C)}$$  \hspace{1cm} (34)

and where we have defined the non-dimensional collapse number, $C$, as

$$C = \frac{\rho_cgh_o\lambda^2}{16\pi^2Kn\eta}.$$  \hspace{1cm} (35)

The non-dimensional collapse number is a measure of the relative importance of erosion to gravitational collapse. If $C=0$, the characteristic time for combined erosional and gravitational collapse defined in Eq. (34) reduces to the characteristic time for erosion defined in Eq. (7). Alternatively, if $C$ approaches infinity, Eq. (34) reduces to the characteristic time for gravitational collapse defined in Eq. (25). Comparing Eqs. (7) and (34) we see that characteristic time for combined erosional and gravitational collapse, $\tau_{eg}$, can be related to the characteristic time for erosion, $\tau_e$, by

$$\tau_{eg} = \frac{\tau_e}{1+C}.$$  \hspace{1cm} (36)

We now obtain a solution for the total crustal thickness, $h(x,t)$, for the combined erosional and gravitational collapse case by substitution of Eq. (33) into Eq. (19) and then substituting this result into Eq. (15)

$$h(x,t) = (h_1-h_o)\exp\left(-\frac{t}{\tau_{eg}}\right)\cos\left(\frac{2\pi x}{\lambda}\right) + h_o.$$  \hspace{1cm} (37)

Substitution of Eq. (37) into Eq. (3) yields an expression for the height of topography

$$w(x,t) = w_1\exp\left(-\frac{t}{\tau_{eg}}\right)\cos\left(\frac{2\pi x}{\lambda}\right).$$  \hspace{1cm} (38)

Assuming this topography is harmonic with a wavelength, $\lambda$, we find an exponential decay of the topography in time. Substitution of Eq. (33) into Eq. (22) and the result into Eq. (20), and using the definition in Eq. (16), yields the following expression for the horizontal velocity,

$$u(x,t) = (h_1-h_o)\frac{\rho_cgh_o\lambda}{8\pi\eta}\exp\left(-\frac{t}{\tau_{eg}}\right)\sin\left(\frac{2\pi x}{\lambda}\right).$$  \hspace{1cm} (39)

The forms of Eqs. (37)–(39) are the same as those given in Eqs. (26)–(28), however the characteristic time for the case of gravitational collapse alone and that for the combined erosional and gravitational collapse are different.

The maximum surface relaxation velocity, $v_{emax}$, due to both erosion and gravitational collapse, at $x=0$ and $t=0$ is obtained from Eq. (38) with the result

$$v_{emax} = -\frac{\partial w}{\partial t}\big|_{x=0,t=0} = \frac{w_1}{\tau_{eg}}.$$  \hspace{1cm} (40)

The maximum erosion rate is obtained from Eqs. (30) and (38) with the result

$$v_{emax} = -K\frac{\partial^2 w}{\partial x^2}\big|_{x=0,t=0} = \frac{4\pi^2Kw_1}{\lambda^2}.$$  \hspace{1cm} (41)

and from Eqs. (40) and (41) we have

$$v_{emax} = \left(1-\frac{\rho_c}{\rho_m}\right)^{-1}\frac{v_{emax}}{(1+C)}.$$  \hspace{1cm} (42)

In the limit as $C$ approaches infinity (gravitational collapse only), we have $v_{emax}=0$ and in the limit as $C=0$ (erosion only), Eq. (42) reduces to Eq. (12) as expected.

Lastly, the total depth of erosion, $h_e$, as a function of the initial topography is

$$h_e = \frac{w_1}{\left(1-\frac{\rho_c}{\rho_m}\right)(1+C)}.$$  \hspace{1cm} (43)
In the limit as \( C \) approaches infinity (gravitational collapse only), \( h_e=0 \). In the limit as \( C=0 \) (erosion only), \( h_e=h_1-h_o \). This is consistent with results shown earlier, that is, if \( C=0 \) the characteristic time for combined erosional and gravitational collapse reduces to the characteristic time for erosional collapse.

The non-dimensional collapse number, \( C \), defined in Eq. (35), depends on the erosion coefficient, \( K \), and the viscosity of the crust, \( \eta \), as well as on \( \lambda \), \( \rho_c \), and \( h_o \). \( K \) is a measure of the erosion rate, and \( \eta \) is a measure of the thinning of the crust due to buoyancy driven lateral flow. Thus, the relative importance of erosion and gravitational collapse in the removal of topography can be quantified by comparison of the erosion coefficient and the crustal viscosity. Taking the same values for \( \lambda \), \( \rho_c \), and \( h_o \) as in previous sections, we determine the dependence of the erosion coefficient, \( K \), on the crustal viscosity, \( \eta \), from Eq. (35) for a range in values of \( C \), and show the results in Fig. 6. Above the line \( C=1 \), i.e., for values of \( K \) and \( \eta \) that correspond to values of the non-dimensional collapse number that are less than one, erosion is the dominant mechanism. Below the curve \( C=1 \), i.e., for values of \( K \) and \( \eta \) that correspond to values of the collapse number that are greater than one, gravitational collapse will dominate. Fig. 6 indicates that for small values of crustal viscosity (on the order of \( 1 \times 10^{21} \) Pa s), large values of \( K \) are required for erosional processes to dominate, whereas, for large values of crustal viscosity (on the order of \( 1 \times 10^{23} \) Pa s), much smaller values of \( K \) are permissible for erosion to be the predominant mechanism in the removal of topography.

We now use the expression for the collapse number defined in Eq. (35) and the expression for the maximum erosion rate, \( v_{\text{emax}} \), defined in Eq. (41) to calculate the dependence of the maximum erosion rate on the viscosity for several values of the initial topographic height of a plateau, \( w_1 \), assuming \( C=1 \). We show the resulting dependence of the maximum erosion rate on the viscosity in Fig. 7, assuming the same values of \( \lambda \), \( \rho_c \), and \( h_o \) as in the previous examples. Note that although each line in Fig. 7 corresponds to a particular initial topographic height of a plateau, all lines correspond to the case \( C=1 \). Therefore, for a given line, the combinations of \( v_{\text{emax}} \) and \( \eta \) located above the line correspond to the situation where the erosion is the dominant mechanism in the removal of topography, whereas, the combinations of \( v_{\text{emax}} \) and \( \eta \) located below the line correspond to the situation where gravitational collapse is the dominant mechanism in the decay of the topography. For a plateau or collisional orogen with a low viscosity (on the order of \( 1 \times 10^{21} \) Pa s), erosion rates on the order of 10–30 mm/yr are required for erosion to be the dominant mechanism in the decay of the topography, depending on the initial topographic height, \( w_1 \). In contrast, if the viscosity of the plateau or collisional orogen is large (on the order of \( 1 \times 10^{23} \) Pa s), erosion rates between 0.1–0.5 mm/yr are permissible for erosion to be the predominant mechanism, depending on the initial topographic height, \( w_1 \). The greater the average surface elevation of the plateau, the greater the relative erosion rate and viscosity required for erosion to be the predominant mechanism.

We again set \( C=1 \) in Eq. (35), but now choose specific values for the maximum erosion rate and topographic height in order to solve for the specific values of \( K \) and \( \eta \) that correspond to a plateau of comparable scale as the Tibetan plateau to be in the transitional situation where both erosion and gravitational collapse are equally important, i.e., for the case where \( C=1 \). Using an initial velocity of erosion, \( v_{\text{emax}} = 0.001 \) mm/yr, an initial topographic amplitude of 5 km, and the same values of \( \lambda \), \( \rho_c \), \( \rho_m \), and \( h_o \) as in the previous examples, we find from Eq. (41) that \( K \) is equal to \( 4.1 \times 10^4 \) m³/yr. Substituting this value of \( K \) and the same values of \( \lambda \), \( \rho_c \), and \( h_o \) as before into Eq. (35) we find that the predicted value of viscosity is \( 1.1 \times 10^{22} \) Pa s. Thus, if the equivalent crustal viscosity is greater than this, erosion is expected to dominate over buoyancy driven flow of the crust. For smaller viscosities buoyancy driven flow will dominate.
Previous estimates for the effective viscosity of the Tibetan lithosphere are $5 \times 10^{21} - 2 \times 10^{22}$ Pa s, for strain rates of $10^{-15} - 10^{-16}$ s$^{-1}$ (England and Molnar, 1997), and $0.5 \times 10^{22} - 5 \times 10^{22}$ Pa s (Flesch et al., 2001). Comparison of these estimates with the value of viscosity, $1.1 \times 10^{22}$ Pa s, predicted above for the case where $C=1$ and $w_1=5$ km suggests that we cannot conclude definitively that one mechanism is dominant over the other for loss of topography associated with the future decay of the Tibetan Plateau. Note that the predicted viscosity of $1.1 \times 10^{22}$ Pa s corresponds to a maximum erosion rate of 1 mm/yr, however, estimates for the maximum erosion rate in the Tibetan plateau and Himalaya vary (Fielding, 1996; Burbank, 2002; Burbank et al., 2003). The line $w_1=5$ km in Fig. 7 shows how the viscosity, for the case where $C=1$, predicted by our model varies with the maximum erosion rate for a plateau of comparable scale as the Tibetan plateau. The prediction that a viscosity of less than $1.1 \times 10^{22}$ Pa s is required for gravitational collapse to be the dominant mechanism in the removal of topography of the Tibetan plateau, based on the assumptions above, is consistent with the results of Liu and Yang (2003) that suggest effective viscosities on the order of $10^{21} - 10^{22}$ Pa s are required for extensional collapse of the Tibetan plateau.

6. Conclusions

After the end of a mountain building event, the topography associated with the orogeny is reduced over a period of time. Two basic mechanisms have been proposed for the loss of this topography:

(1) Erosion. Erosion will certainly result in the removal of material from high elevations and the transport of material to lower elevations.

(2) Gravitational collapse. Elevated topography is gravitationally unstable and associated body forces will result in lateral flows and loss of topography.

It is the purpose of this paper to obtain an analytical solution that quantifies the relative importance of these two mechanisms in the removal of topography. In doing this we make many assumptions and reduce the problem to two parameters: the transport coefficient, $K$, and the viscosity, $\eta$.

In quantifying erosion we use the Culling model. We have discussed this approach in detail. Although the Culling model can be criticized for its simplicity (mass movement is proportional to slope), no alternative comprehensive model has been proposed for use in an analytic model. We consider the value of the transport coefficient, $K$, to be a measure of the erosion rate. Clearly it will have a strong dependence on many factors including rock type, climate, and surface morphology. Glacial effects will also affect the erosion rate.

We apply the thin viscous sheet approximation to model gravitational collapse. There are many alternatives for quantifying the buoyancy driven flow in the crust and mantle due to gravitational collapse. However, these generally are reduced to either a plastic or a fluid rheology. We assume a linear viscous rheology and that the thickness of the continental crust is equivalent to the thickness of the highly viscous lithosphere. This assumption could be modified by assuming that flows take place in a lithosphere with an prescribed thickness but our results would not be significantly affected and an additional parameter is required. We consider the viscosity to be a measure of crustal flow just as the transport coefficient is a measure of material removal due to erosion. Although we do not include temperature in our calculations, we expect that the thermal state of the lithosphere is likely to play an important role in crustal flows. As the lithosphere cools and thickens following an orogeny, the equivalent viscosity will increase and the relative role of erosion is expected to increase. Alternatively, if the lower mantle lithosphere is convectively thinned, the temperature at the base of the crust may increase and, in that case, the relative role of gravitational collapse may dominate

![Fig. 7. Dependence of the maximum erosion rate, $v_{\text{max}}$, on the viscosity of the crust, $\eta$, for a range in the initial height of topography, $w_1$, assuming $C=1$. Erosion is dominant above the lines, and gravitational collapse is dominant below the lines.](image-url)
(Sandiford and Powell, 1990; Zhou and Sandiford, 1992).

To further simplify our analysis, we make several additional assumptions. We linearize the fluid equations by assuming that the change in crustal thickness is small compared to its reference value. In addition, the initial topography is assumed to be harmonic with wavelength, $\lambda$. Since our problem is linear, any prescribed distribution of topography can be considered by superposition. We assume two-dimensionality, but again, because of the linearity any arbitrary harmonic three-dimensional topography can be considered by superposition. We assume the topography is isostatically compensated. Generally, orogenic plateaus and collisional orogens are sufficiently broad for Airy isostasy to be a good approximation.

Considering only erosion we find that characteristic erosion times in the range of $10^7 \leq \tau_e \leq 10^8$ yr require transport coefficients in the range of $6.7 \times 10^4 \geq K \geq 6.7 \times 10^3$ m$^2$/yr. Considering only buoyancy driven flows we find that the characteristic time in the range of $10^7 \leq \tau_g \leq 10^8$ requires viscosities in the range of $1.3 \times 10^{-22} \leq \eta \leq 1.3 \times 10^{23}$ Pa s. We have also considered the simultaneous roles of erosion and gravitational collapse. Our results give an exponential decay of the topography in time, and we find that the relative role of erosion versus gravitational collapse for compensated topography is controlled by the value of the non-dimensional collapse number $C = \frac{gh_o \lambda^2 \rho_c}{16 Kn \pi^2}$. If $C$ is large compared with unity gravitational collapse dominates, if $C$ is small compared with unity erosion dominates. We conclude that both erosion and crustal flows are acceptable mechanisms for loss of topography following an orogeny. Considering the combined erosion and gravitational collapse case and using parameter values representative of the Tibetan plateau, and assuming $C=1$, i.e., that erosion and gravitational collapse are equally operative, the corresponding predicted value of viscosity is on the order of $1.1 \times 10^{22}$ Pa s. This value is typical of values associated with the effective viscosity of the Tibetan lithosphere (England and Molnar, 1997; Flesch et al., 2001). Therefore, we cannot conclude definitively that one mechanism is dominant over the other for loss of topography associated with the future decay of the Tibetan Plateau.

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