Assignment 2 for GEOL 1820: Geophysical Fluid Dynamics, Waves and Mean Flows Edition Due Oct. 17, 2016

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Contacts

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Getting Help!

I am usually available by email. You can make an appointment other times. Just check my calendar at http://fox-kemper.com/contact and suggest a time that works for you.

1 Problem 1: Parabolic Ray Focusing

Do problem 3.4.2 of (Bühler, 2014).

2 Problem 2: Reconciling Wave Flux with Energy and Group Velocity

In the WKB expansion of (Bühler, 2014), we have the ansatz (i.e., solution guess) and equations,

$$h = A(x, y)e^{i(\kappa_0 s(x,y) - \alpha - \omega t)} \tag{1}$$

$$|\nabla s|^2 = n^2, \tag{2}$$

$$\nabla \cdot \left(\frac{A^2}{n^2} \nabla s \right) = 0. \tag{3}$$

In the phase velocity, group velocity slides (see also Chp. 1 of Chapman & Rizzoli, 1989)), it is discussed how energy propagates with the group velocity.

$$\mathbf{c}_g = \frac{\partial \omega}{\partial k} \hat{\mathbf{k}} \tag{4}$$

Relate these two frameworks to express the argument of the divergence in (3) as a group velocity times a conserved quantity. What is the conserved quantity? How do you relate $\frac{\partial \omega}{\partial k}$ to $s(x, y)$? Hints: Here ω doesn't vary k does, and $c_g = c_p$ for shallow water waves.

3 Comparison of Stationary Phase

Compare section 3.2.3 of Bühler (2014) with section 1.4 of Chapman & Rizzoli (1989). What is the essence of the method of stationary phase?

4 Phase and Group Velocity

Find the phase speed in k, ℓ directions and the group velocity (vector, gradient w.r.t. k, ℓ) for the following dispersion relations for x wavenumber k and y wavenumber ℓ. $(k, l) = \boldsymbol{\kappa}$, and $\kappa = |\boldsymbol{\kappa}|$. Subscript 0 indicates a constant. Hint: $\frac{\partial \kappa}{\partial k} = \frac{k}{\kappa}, \frac{\partial \kappa}{\partial \ell} = \frac{\ell}{\kappa}$

$$\omega = \mathbf{c}_0 \cdot \boldsymbol{\kappa} \tag{5}$$

$$\omega^2 = \mathbf{c}_0^2 \, \kappa^2 \tag{6}$$

$$\omega^2 = gH\kappa^2 \quad \text{shallow-water waves} \tag{7}$$

$$\omega^2 = g\kappa \quad \text{deep-water waves} \tag{8}$$

$$\omega = \frac{-\beta k}{\kappa^2} \quad \text{Rossby waves} \tag{9}$$

5 Piecewise Beach

Consider the following index of refraction variations:

$$n(x, y) = \begin{cases} n_d & x \leq -L \\ x\frac{n_s - n_d}{2L} + \frac{n_s + n_d}{2} & |x| \leq |L| \\ n_s & x \geq L \end{cases} \tag{10}$$

5.1

For shallow water waves, how does the depth vary over this region?

5.2

For waves that are directly incident on the slope (i.e, $s(x, y) = x$ and $A = 1$ at $x \ll -L$), solve for $A(x, y)$ and $s(x, y)$ using equations (3.6) and (3.8) of Bühler (2014).

5.3

For waves that hit the slope obliquely (i.e, $s(x, y) = x\cos\theta_0 + y\sin\theta_0$ and $A = 1$ at $x \ll -L$), solve for $A(x, y)$ and $s(x, y)$ using equations (3.6) and (3.8) of Bühler (2014).

References

Bühler, Oliver 2014 *Waves and mean flows*, 2nd edn. Cambridge, United Kingdom: Cambridge University Press.

Chapman, David C. & Rizzoli, Paola M. 1989 Wave motions in the ocean: Myrl's view. *Tech. Rep.*. MIT/WHOI Joint Program, Woods Hole, Mass.