Assignment 3 for GEOL 1820: Geophysical Fluid Dynamics, Waves and Mean Flows Edition Due Oct. 17, 2016

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Contacts

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Getting Help!

I am usually available by email. You can make an appointment other times. Just check my calendar at http://fox-kemper.com/contact and suggest a time that works for you.

1 Rossby waves created by sidewall undulations

Do problem 4.7.1 of (?).

2 RSWE Waves

Begin with the rotating shallow water equations.

$$\frac{D\mathbf{u}}{Dt} + \mathbf{f} \times u + g\nabla\eta, \tag{1}$$

$$\frac{Dh}{Dt} + h\nabla \cdot \mathbf{u} = 0. \tag{2}$$

Linearize about zero background flow, assuming a flat bottom with constant Coriolis parameter $\mathbf{f} = f_0\hat{\mathbf{z}}$. Derive the plane wave dispersion relation.

$$\omega^2 = f^2 + c^2\kappa^2. \tag{3}$$

Section 2.1.4 may be a helpful guide.

3 QG Waves

Begin with the quasigeostrophic equations on the β-plane

$$\frac{Dq}{Dt} = 0, \tag{4}$$

$$q = \nabla^2\psi - \kappa_D^2\psi + \beta y, \tag{5}$$

$$u = -\psi_y, \qquad v = \psi_x, \tag{6}$$

$$\psi = \frac{gh}{f_0}, \qquad \kappa_D = \frac{f}{\sqrt{gH}} \tag{7}$$

Linearize about zero background flow, assuming a flat bottom. Derive the plane Rossby wave dispersion relation:

$$\omega = \frac{-\beta k}{\kappa^2 + \kappa_D^2} \tag{8}$$

4 Plotting

Plot the dispersion relations from the RSWE and QG solutions above on the same figure. Calculate (numerically) the frequency, phase, and group speeds of RSWE waves (assuming $gH = 10000 m^2/s^2$, $f = 10^{-4}$/s) and QG Rossby waves (assuming $f = 10^{-4}$/s, $\beta = 2 \cdot 10^{-11}$/sm, $\kappa_D = 1/100$km) in the following cases:[1]

1. $k = 2\kappa_D, l = 0$. (This is approximately a 314 km wavelength).
2. $k = 0.1/m, l = 0$. (This is approximately a 62 m wavelength–typical size of gravity waves, although it technically violates the shallow water approximation).
3. $k = \kappa_D/10, l = 0$ (This is approximately 1 wavelength/Atlantic span).

How long does it take for a wave packet of each type to cross the Atlantic (6000 km) zonally (i.e., in the k direction)?

5 Zonal Flow

Reconsider the RSWE dispersion relation and the QG dispersion relation by linearizing about a mean zonal flow U instead of a zero background flow. How do the dispersion relations change? (Hint: It is very useful to consider the intrinsic and absolute frequencies in this derivation, see (4.90)).

[1] Note: this value of the deformation wavenumber is much smaller than you'd get for the real ocean. It is similar to that in the real, stratified ocean, or it can be estimated using the *reduced gravity* approximation. See chaper 12 of **?** if you are interested in the details.