Beyond GM
A Symposium on Oceanic Eddy Fluxes

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The Character of the Mesoscale

(Capet et al., 2008)

- Boundary Currents
- Eddies
- \( \text{Ro} = O(0.1) \)
- \( \text{Ri} = O(1000) \)
- Full Depth
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly baroclinic & barotropic instability. Parameterizations of baroclinic instability (GM, Visbeck...).
The Character of the Submesoscale (Capet et al., 2008)

- Fronts
- Eddies
- $\text{Ro}=O(1)$
- $\text{Ri}=O(1)$
- near-surface
- 1–10km, days

Eddy processes mainly include baroclinic instability (Boccaletti et al. ’07, Haine & Marshall ’98).

Parameterizations of baroclinic instability apply? (GM, Visbeck...).
Tracer Flux-Gradient Relationship

\[ u' \tau' = -M \nabla \tau \]

Most subgridscale eddy closures have this form: GM*, Redi, FFH** submesoscale

Relates the eddy flux to the coarse-grain gradients locally

If we knew the dependence of \( M \) on the coarse-resolution flow, we'd have the optimal local eddy closure

*Gent & McWilliams (1990)  **Fox-Kemper, Ferrari, Hallberg (2008)
Dukowicz & Smith (97) and Smith (99) lay out the form of a stochastic, adiabatic relocation of particles.

The resulting Fokker-Planck Equation for the probability density of the particles gives a $K$ and a $v$, which are closely related to the Lagrangian mean transport and the diffusion of probability.

\[
\frac{\partial p(x, t | y, t_0)}{\partial t} + \nabla \cdot U p(x, t | y, t_0) = \nabla \cdot K \cdot \nabla p(x, t | y, t_0)
\]

\[
U = v - \nabla \cdot K
\]

\[
v(x, t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int dx' (x' - x)p(x', t + \Delta t | x, t)
\]

\[
K(x, t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int dx' \frac{1}{2} (x' - x) (x' - x)p(x', t + \Delta t | x, t).
\]

Since the pdf is positive, it is clear from (53) that $K$ is a $2 \times 2$ symmetric positive-definite tensor. We will refer to $v$ as the Lagrangian mean velocity, although this identification is not exact (see Bennett 1996, p. 7). In (52)
\[ \begin{bmatrix} u' \tau' \\ v' \tau' \\ w' \tau' \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \]

Small slope approximation converts the horizontal stochastic reshuffling along neutral surfaces to a GM+Redi like form.

Could vary tracer by tracer, or active tracer vs. passive, etc. In practice we don’t let it.

Using the same form for all tracers amounts to ‘labeling’ fluid with tracer, neglecting sources, etc.
\[ \underline{u'} \cdot \underline{\tau'} = -\underline{M} \nabla \underline{\tau} \]

**Sym Part=Anisotropic**

Redi (1982) are symmetric and scaled to make eddy mixing along neutral surfaces.

\[
\begin{bmatrix}
\underline{u'} \cdot \underline{\tau'} \\
\underline{v'} \cdot \underline{\tau'} \\
\underline{w'} \cdot \underline{\tau'}
\end{bmatrix} = -
\begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot K \cdot \hat{\nabla} z \\
K_{yx} & K_{yy} & \hat{y} \cdot K \cdot \hat{\nabla} z \\
\hat{x} \cdot K \cdot \hat{\nabla} z & \hat{y} \cdot K \cdot \hat{\nabla} z & \hat{\nabla} z \cdot K \cdot \hat{\nabla} z
\end{bmatrix}
\begin{bmatrix}
\tau_x \\
\tau_y \\
\tau_z
\end{bmatrix}
\]

**Yellow** \( K \) ‘are’ horizontal stirring & mixing.

**Blue** factors in Redi (1982) are symmetric and scaled to make eddy mixing along neutral surfaces.

*Anisotropic form due to Smith & Gent 04
AntiSym Part = Anisotropic\* GM

\[ \overline{u'\tau'} = -\overline{M \nabla \tau} \]

**Antisymmetric Elements in GM (1990)**

\[
\begin{bmatrix}
\overline{u'\tau'} \\
\overline{v'\tau'} \\
\overline{w'\tau'}
\end{bmatrix} = -
\begin{bmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \mathbf{\nabla} z \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \mathbf{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \mathbf{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \mathbf{\nabla} z & 0
\end{bmatrix}
\begin{bmatrix}
\overline{\tau_x} \\
\overline{\tau_y} \\
\overline{\tau_z}
\end{bmatrix}
\]

Antisymmetric Elements in GM (1990) are scaled to overturn fronts, make vertical fluxes extract PE, and restratify the fluid equivalent to eddy-induced advection.

**Q:** Same horiz. mixing (\(\mathbf{K}\)) as Redi?

*Anistropic form due to Smith & Gent 04  \*Tensor Form (Griffies, 98)
An example of what a (submeso) subgrid parameterization looks like with FLOW DEPENDENT $\mathbf{M}$:


$$\begin{bmatrix}
u'\tau' \\
v'\tau' \\
w'\tau'
\end{bmatrix} = -
\begin{bmatrix}
0 & 0 & -\Psi_y \\
0 & 0 & \Psi_x \\
\Psi_y & -\Psi_x & 0
\end{bmatrix}
\begin{bmatrix}
\Psi_x \\
\Psi_y \\
\Psi_z
\end{bmatrix}
$$

$$\Psi = \begin{bmatrix}
\Delta x \\
\frac{L_f}{C_e H^2 \mu(z) \sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{z}
\end{bmatrix}
$$

$$\mu(z) = \begin{bmatrix}
1 - \left(\frac{2z}{H} + 1\right)^2 \\
1 + \frac{5}{21} \left(\frac{2z}{H} + 1\right)^2
\end{bmatrix}$$
\[ \mathbf{u}' \mathbf{\tau}' = - \mathbf{M} \nabla \mathbf{\tau} \]

**Sym Part = Anisotropic**

\[
\begin{bmatrix}
\mathbf{u}' \mathbf{\tau}' \\
\mathbf{v}' \mathbf{\tau}' \\
\mathbf{w}' \mathbf{\tau}'
\end{bmatrix} = - \begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \mathbf{\nabla} z \\
K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \mathbf{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \mathbf{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \mathbf{\nabla} z & \mathbf{\nabla} z \cdot \mathbf{K} \cdot \mathbf{\nabla} z
\end{bmatrix} \begin{bmatrix}
\mathbf{\tau}' x \\
\mathbf{\tau}' y \\
\mathbf{\tau}' z
\end{bmatrix}
\]

**AntiSym Part = Anisotropic**

\[
\begin{bmatrix}
\mathbf{u}' \mathbf{\tau}' \\
\mathbf{v}' \mathbf{\tau}' \\
\mathbf{w}' \mathbf{\tau}'
\end{bmatrix} = - \begin{bmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \mathbf{\nabla} z \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \mathbf{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \mathbf{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \mathbf{\nabla} z & 0
\end{bmatrix} \begin{bmatrix}
\mathbf{\tau}' x \\
\mathbf{\tau}' y \\
\mathbf{\tau}' z
\end{bmatrix}
\]

Yellow \( \mathbf{K} \) 'are' horizontal stirring & mixing
Need a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{align*}
\overline{u'\tau'} & = - \mathbf{M} \nabla \overline{\tau} \\
\begin{bmatrix}
\overline{u'\tau'} \\
\overline{v'\tau'} \\
\overline{w'\tau'}
\end{bmatrix} & = -
\begin{bmatrix}
M_{xx} & M_{xy} & M_{xz} \\
M_{yx} & M_{yy} & M_{yz} \\
M_{zx} & M_{zy} & M_{zz}
\end{bmatrix}
\begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\end{align*}
\]

3 equations/tracer

9 unknowns (\(\mathbf{M}\) components)

BY USING 3 or MORE TRACERS, can determine \(\mathbf{M}\)!!!

(a la Plumb & Mahlman '87, Bratseth '98)

No assumptions about symmetry required.
Need a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{align*}
\overline{u'\tau'} &= -M \nabla \overline{\tau} \\
\begin{bmatrix}
\overline{u'\tau'} \\
\overline{v'\tau'} \\
\overline{w'\tau'}
\end{bmatrix} &= -
\begin{bmatrix}
M_{xx} & M_{xy} & M_{xz} \\
M_{yx} & M_{yy} & M_{yz} \\
M_{zx} & M_{zy} & M_{zz}
\end{bmatrix}
\begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\end{align*}
\]

With John Dennis & Frank Bryan, we took a POP0.1° Normal-Year forced model (yrs 16-20 for anal.)

Added 9 Passive tracers--restored to x,y,z @ 3 rates

Kept all the eddy fluxes for passive & active tracers

Coarse-grained to 2°, passive tracers to find M
Could you have guessed it?
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\bar{u}'\tau' \\
\bar{v}'\tau' \\
\bar{w}'\tau'
\end{bmatrix}
= - \begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \hat{\nabla}z \\
K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla}z \\
\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla}z & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla}z & \hat{\nabla}z \cdot \mathbf{K} \cdot \hat{\nabla}z
\end{bmatrix}
\begin{bmatrix}
\bar{\tau}_x \\
\bar{\tau}_y \\
\bar{\tau}_z
\end{bmatrix}
\]

Hor. Diffusivity is roughly Trace(M)/2

Peak of Diffusivity near 250 m^2/s

Median Diffusivity near 1000 m^2/s

<6% negative
Interpretation?

- Isoneutral diffusion or ‘mixing’: symmetric $\mathbf{K}$ with real, positive eigenvalues (neg→nonlocal)

- The eigenvalues of $\mathbf{M}$ are related, except there is one more involving the neutral to $z$ coordinate conversion (in S&G theory, at least)

- The eigenvectors give the direction of the mixing associated with each eigenvalue

- Antisymmetric $\mathbf{K}$ & $\mathbf{M}$ are stirring/overturning by an eddy-induced (quasi-stokes) streamfunction--non-orthogonal eigenvects and imaginary eigenvalues possible!
Result: Strong Anisotropy Along/Across Isopycnals

Mixing:

Stirring:
Result: \( K = GM K \) (mostly)

If so these 2 components should match in Sym & Antisym M
Result: Redi $K = GM_K$ (mostly)

If so these 2 components should match in Sym & Antisym $M$
Result: Strong Anisotropy Along/Across PV Grads.

Mixing:

Stirring:
FIG. 12. Inferred horizontal eddy diffusivity $\kappa$ (m$^2$ s$^{-1}$): (top) zonal mean and (bottom) vertical mean over the thermocline (0–1200 m). The contour intervals are (top) 500 and (bottom) 1000 m$^2$ s$^{-1}$. The thick line indicates the zero contour. Also indicated in the bottom panel are the 10-, 70-, and 130-Sv contours of the barotropic streamfunction.

Comparisons with Marshall et al.

Realistic negative eigs. vs. spurious?
Comparisons with Marshall et al.

Ferreira, Marshall, Heimbach 05

Zonal mean (scalar) diffusivity

vs.

Eigenvalues of $M$

Same shape--few negatives!
How do we explain the Horizontal Variations of $K$?
Eden & Greatbatch (+others) propose that baroclinic instability’s production of EKE from PE should guide M magnitude.
Locations of large eigs of $K$
Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations

![Figure 1](image)

**Fig. 1.** Annual mean thickness diffusivity ($K$) in m$^2$/s at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of $K$ are own for the interior region only, i.e. values of $K$ in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear colour scale for the thickness diffusity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The red mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model's land mask.
Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations

**Fig. 1.** Annual mean thickness diffusivity \((K)\) in \(m^2/s\) at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of \(K\) are shown for the interior region only, i.e. values of \(K\) in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear colour scale for the thickness diffusivity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The land mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model’s land mask.
How do we explain the Vertical Variations of K?
Result:
course KE→ vertical structure of Mixing

\[ K \propto \sqrt{\langle KE \rangle} \]

Even better with EKE!
Note--barotropic mode is in there!

Monday, June 13, 2011
Comparisons with Marshall et al.

Abernathy et al. 09

Monday, June 13, 2011
Comparisons with Marshall et al.

Critical Layer?
thus nonlocal vert. modes?

Abernathy et al 09

Monday, June 13, 2011
Conclusions

A method for diagnosing the eddy stirring associated with fluxes represented in a 0.1° model but not a 2° model is presented.

It estimates the tracer-type-independent transport of tracer.

The shape and structure agrees roughly with Griffies (98) and Gent & Smith (04) analyses of GM & Redi isoneutral fluxes with *equal* anisotropic mixing & stirring.

No gauge/rot. fluxes are needed to eliminate negative spurious eigenvalues.
Use a Natural, Mesoscale Eddy Environment to Test Out:

Testing the **Diagnosis:**

Note: T not used for diagnosis, active tracers are apparently transported as passive ones are!
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\underline{u'\tau'} \\
\underline{v'\tau'} \\
\underline{w'\tau'}
\end{bmatrix} = -\begin{bmatrix}
0 & 0 & -\hat{x} \cdot K \cdot \tilde{\nabla}z \\
0 & 0 & -\hat{y} \cdot K \cdot \tilde{\nabla}z \\
\hat{x} \cdot K \cdot \tilde{\nabla}z & \hat{y} \cdot K \cdot \tilde{\nabla}z & 0
\end{bmatrix} \begin{bmatrix}
\overline{T_x} \\
\overline{T_y} \\
\overline{T_z}
\end{bmatrix}
\]

Result 1: Antisymmetric (GM) Elements scale with corresponding Symmetric (Redi) elements in extratropics.

Thus, GM/Redi basic shape of M is roughly correct
(some detailed validation remains)
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{pmatrix}
\overline{u' \tau'} \\
\overline{v' \tau'} \\
\overline{w' \tau'}
\end{pmatrix}
= -
\begin{pmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\
\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & 0
\end{pmatrix}
\begin{pmatrix}
\overline{\tau_x} \\
\overline{\tau_y} \\
\overline{\tau_z}
\end{pmatrix}
\]

Asym 3.1: GM@z=-149\(\mu\)m

Asym 3.2: GM@z=-149\(\mu\)m

Monday, June 13, 2011
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\bar{u}'\tau' \\
\bar{v}'\tau' \\
\bar{w}'\tau'
\end{bmatrix} = -
\begin{bmatrix}
K_{xx} & K_{xy} & \hat{x}\cdot K\cdot \vec{\nabla}z \\
K_{yx} & K_{yy} & \hat{y}\cdot K\cdot \vec{\nabla}z \\
\hat{x}\cdot K\cdot \vec{\nabla}z & \hat{y}\cdot K\cdot \vec{\nabla}z & \vec{\nabla}z\cdot K\cdot \vec{\nabla}z
\end{bmatrix}
\begin{bmatrix}
\bar{\tau}_x \\
\bar{\tau}_y \\
\bar{\tau}_z
\end{bmatrix}
\]
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\overline{u'} \overline{\tau'} \\
\overline{v'} \overline{\tau'} \\
\overline{w'} \overline{\tau'}
\end{bmatrix} = - \begin{bmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \vec{\nabla} z \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \vec{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \vec{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \vec{\nabla} z & 0
\end{bmatrix} \begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\]

Atlantic Section
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\bar{u}' \bar{\tau}' \\
\bar{v}' \bar{\tau}' \\
\bar{w}' \bar{\tau}'
\end{bmatrix}
= -
\begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{\nabla} z \cdot \mathbf{K} \cdot \hat{\nabla} z
\end{bmatrix}
\begin{bmatrix}
\bar{\tau}_x \\
\bar{\tau}_y \\
\bar{\tau}_z
\end{bmatrix}
\]
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\overline{u'} \overline{\tau'} \\
\overline{v'} \overline{\tau'} \\
\overline{w'} \overline{\tau'} \\
\end{bmatrix} = -
\begin{bmatrix}
0 & 0 & -\overline{x} \cdot \mathbf{K} \cdot \mathbf{\bar{\nabla}} z \\
0 & 0 & -\overline{y} \cdot \mathbf{K} \cdot \mathbf{\bar{\nabla}} z \\
\overline{\mathbf{x}} \cdot \mathbf{K} \cdot \mathbf{\bar{\nabla}} z & \overline{\mathbf{y}} \cdot \mathbf{K} \cdot \mathbf{\bar{\nabla}} z & 0 \\
\end{bmatrix}
\begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z \\
\end{bmatrix}
\]

Pacific Section (180E)

Monday, June 13, 2011
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
    u' \tau' \\
    v' \tau' \\
    w' \tau'
\end{bmatrix}
= -
\begin{bmatrix}
    K_{xx} & K_{xy} & \hat{x} \cdot K \cdot \nabla z \\
    K_{yx} & K_{yy} & \hat{y} \cdot K \cdot \nabla z \\
    \hat{x} \cdot K \cdot \nabla z & \hat{y} \cdot K \cdot \nabla z & \nabla z \cdot K \cdot \nabla z
\end{bmatrix}
\begin{bmatrix}
    \overline{\tau}_x \\
    \overline{\tau}_y \\
    \overline{\tau}_z
\end{bmatrix}
\]
NSEF & Diabatic/Transition Layer

Danabasoglu & Marshall

Danabasoglu, Ferrari & McWilliams

Ferrari, McWilliams, Canuto, Dubovikov

Surface-intensified GM, no boundary condition issues, no over-restratification of Mixed Layer by Eddies

Fig. 2. A conceptual model of eddy fluxes in the upper ocean. Mesoscale eddy fluxes (blue arrows) act to both move isopycnal surfaces and stir materials along them in the oceanic interior, but the fluxes become parallel to the boundary and cross density surfaces within the BL. Microscale turbulent fluxes (red arrows) mix material vertically on a characteristic eddy turnover time in the interior.
Near-surface eddy flux scheme (Ferrari, McWilliams, Canuto, Dubovikov)

EDDY-INDUCED MERIDIONAL OVERTURNING (GLOBAL)

Vertical profile of zonally-integrated total advective heat flux at 49°S
A new eddy parameterization (Ferrari, Griffies, Nurser & Vallis)

- The eddy streamfunction is given by the elliptic problem

\[ \left( c^2 \frac{d^2}{dz^2} - N^2 \right) \tilde{\Psi} = -\kappa \nabla b \]
\[ \tilde{\Psi} = 0, \quad z = 0, -H \]

Properties of the new parameterization
- releases mean available potential energy
- the eddy transport vanishes at the ocean boundaries
- the eddy transport is dominated by the first baroclinic mode (if $c$ is set to speed of first baroclinic mode)
- does not require any tapering function
- reduces to GM for $c=0$