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Tracer Flux-Gradient Relationship

\[ u' \tau' = -M \nabla \tau \]

Most subgridscale eddy closures have this form: GM*, Redi, FFH** submesoscale

Relates the eddy flux to the coarse-grain gradients locally

If we knew the dependence of \( M \) on the coarse-resolution flow, we’d have the optimal local eddy closure

*Gent & McWilliams (1990)  **Fox-Kemper, Ferrari, Hallberg (2008)
The Character of the Submesoscale

(Capet et al., 2008)

- Fronts
- Eddies
- $Ro = O(1)$
- $Ri = O(1)$
- near-surface
  - 1–10km, days

Eddy processes mainly

baroclinic instability

(Boccaletti et al ’07, Haine & Marshall ’98).

Parameterizations of

baroclinic instability

apply? (GM, Visbeck, FFH).
A Global Parameterization of Mixed Layer Eddy
Restratification

with FLOW DEPENDENT \( \mathbf{M} \) validated against simulations

Fox-Kemper, Ferrari, & Hallberg (2008) &
Fox-Kemper, Danabasoglu, Ferrari, & Hallberg (2008)

\[
\begin{bmatrix}
\overline{u'}\overline{\tau'} \\
\overline{v'}\overline{\tau'} \\
\overline{w'}\overline{\tau'}
\end{bmatrix} = - \begin{bmatrix}
0 & 0 & -\Psi_y \\
0 & 0 & \Psi_x \\
\Psi_y & -\Psi_x & 0
\end{bmatrix} \begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\]

\[
\Psi = \begin{bmatrix}
\Delta x \\
\frac{L_f}{2z + 1}
\end{bmatrix} \cdot \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \cdot \nabla \overline{b} \times \hat{\mathbf{z}}
\]

\[
\mu(z) = \begin{bmatrix}
1 - \left( \frac{2z}{H} + 1 \right)^2 \\
1 + \frac{5}{21} \left( \frac{2z}{H} + 1 \right)^2
\end{bmatrix}
\]

Tuesday, August 3, 2010
Physical Sensitivity of Ocean Climate to Submesoscale Eddy Restratification:

FFH implemented in CCSM (NCAR), CM2M & CM2G (GFDL)

NO RETUNING NEEDED!!!

Improves CFCs

Deep ML Bias reduced

From Fox-Kemper et al., in prep

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Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC

Affects sea ice

NO RETUNING NEEDED!!!

These are impacts: bias change unknown

Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM+ minus CCSM−): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

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Conclusions
Submesoscale

- FFH is implemented in at least 3 IPCC models
- Parameterization reduces bias in CFCs & Mixed Layer Depth
- Parameterization also affects ice & AMOC variability—need truth?
- Flow-dependent, nondimensional scalings validated against simulations *did not require retuning*
The Character of the Mesoscale

(Capet et al., 2008)

- Boundary Currents
- Eddies
- $Ro = O(0.1)$
- $Ri = O(1000)$
- Full Depth
- Eddies strain to produce Fronts
- 100km, months

Eddy processes still baroclinic & barotropic instability.

Parameterizations (GM, Visbeck, Eden).
Virtually all subgridscale eddy closures may be written as: GM, Redi, FFH Submesoscale

Relates the eddy flux to the coarse-grain gradients \( \nabla \tau \) locally.

If we knew the dependence of \( M \) on the coarse-resolution flow, we’d have the optimal local eddy closure.
\[ \begin{bmatrix} \frac{\partial u'}{\partial x} \\ \frac{\partial v'}{\partial y} \\ \frac{\partial w'}{\partial z} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix} \]

Assume same \( \mathbf{M} \) for all tracers:

- 3 equations per tracer
- 9 unknowns (components) + rot-parts (2/tracer)

BY USING 3 or MORE TRACER FLUXES, determine it!!!

(a la Plumb & Mahlman '87, Bratseth '98)
Sym Part=Anisotropic* Redi

\[
\begin{bmatrix}
\bar{u}' \tau' \\
\bar{v}' \tau' \\
\bar{w}' \tau'
\end{bmatrix}
= -
\begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{\nabla} z \cdot \mathbf{K} \cdot \hat{\nabla} z
\end{bmatrix}
\begin{bmatrix}
\bar{\tau}_x \\
\bar{\tau}_y \\
\bar{\tau}_z
\end{bmatrix}
\]

Yellow $\mathbf{K}$ ‘are’ horizontal stirring & mixing

Blue factors in Redi (1982) are symmetric and scaled to make
eddy mixing along neutral surfaces

*Anistropic form due to Smith & Gent 04
\[\begin{align*}
\mathbf{u'} \mathbf{\tau'} &= -\mathbf{M} \nabla \mathbf{\tau'} \\
\end{align*}\]

**AntiSym Part=Anisotropic** GM

\[
\begin{bmatrix}
\mathbf{u'} \mathbf{\tau'} \\
\mathbf{v'} \mathbf{\tau'} \\
\mathbf{w'} \mathbf{\tau'}
\end{bmatrix} = -\begin{bmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z & 0
\end{bmatrix} \begin{bmatrix}
\mathbf{T}_x \\
\mathbf{T}_y \\
\mathbf{T}_z
\end{bmatrix}
\]

Antisymmetric Elements in GM (1990) are scaled to overturn fronts, make vertical fluxes, extract PE, and restratify the fluid equivalent to eddy-induced advection

**Q**: Same horiz. mixing \((\mathbf{K})\) as Redi?

*Anisotropic form due to Smith & Gent 04  
*Tensor Form (Griffies, 98)
Could you have guessed it?
Validation: $\mathbf{M}$ Reproduces T-flux w/o negative eigs.

- Even though Temp not used as tracer to find $\mathbf{M}$

\[
\mathbf{v}'T' = -\mathbf{M} \nabla \overline{T} + O(0.1\% \text{error})
\]

- Typically, diagnoses have problem with $\mathbf{K} < 0$

- Here, below the mixed layer only 6% of gridpoints have negative eigenvalues

- These few negative values are consistent with true nonlocal eddy fluxes
Result: Strong Anisotropy Along/Across Isopycnals

Mixing:

Stirring:

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Result:

Redi $K = GM K$ (mostly)

If so these 2 components should match in Sym & Antisym $M$
Result:

Redi $K = GM K$ (mostly)

If so these 2 components should match in Sym & Antisym $M$
Result: Strong Anisotropy Along/Across PV Grads.

Mixing:

Stirring:
Result: eddy KE→ vertical power law w/ M eigs?

We expect: \( K \propto \sqrt{EKE} \)

But what about: \( K \propto \sqrt{\langle KE \rangle} \)
Result:
coarse KE -> vertical structure of Mixing

$K \propto \sqrt{\langle KE \rangle}$

You don't need to know EKE!
Result:

power law not 'random'

\[ K \propto \sqrt{\langle KE \rangle} \]

However, can probably do better!

Slopes not random.
Coarse-graining--

A matter of philosophy

It would be nicest if when we diagnosed $M$ it agreed with a theory

However, if theory requires, e.g., scale separation, then it likely won’t agree

But, the approach here gives us the answer we need ($M$), even if it’s not the answer we want.

Plumb & Mahlman’s work suffers from the same theoretical issues--McDougall is working on it!
Conclusions Mesoscale

- Direct diagnosis of $M$ is a valuable tool
- Gives validated tracer fluxes without negative eigenvalues or rotational issues
- Still, unfamiliar interpretation
- No clean comparison to theory (GLM? Scale Separation? Ensemble? Stochastics?)
- More to come!

(e.g., Ferrari et al ‘08 vs. Ferrari et al. ‘10)