Eddies, Mixing and all that: Ocean Parameterization Developments from 4m to 400km

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with

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Climate Forecasts (IPCC/CMIP Runs) have a very coarse ocean gridscale (>100km)
Climate Forecasts (IPCC/CMIP Runs) have a very coarse ocean gridscale (>100km)
A Bleeding-Edge Climate Model
(in terms of ocean resolution)
Has some ocean mesoscale instabilities:

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Ocean Equations*: Boussinesq Fluid on Tangent Plane to a Rotating Sphere

\[
\begin{align*}
\partial_t u + u \cdot \nabla_h u + w \partial_z u + \text{Ro}^{-1} f \times u &= -\overline{P} \nabla_h p + \text{Re}^{-1} \nabla^2 u \\
\partial_t w + u \cdot \nabla_h w + w \partial_z w &= -\overline{P} \partial_z p + \Gamma b \hat{z} + \text{Re}^{-1} \nabla^2 w \\
\partial_t b + u \cdot \nabla_h b + w \partial_z b &= \text{Pe}^{-1} \nabla^2 b \\
\nabla_h \cdot u + \partial_z w &= 0
\end{align*}
\]

Buoyancy (or S, T):

Re, Pe for an affordable gridscale are $10^6$ to $10^{11}$

Numerics require O(1)

*From Grooms, Julien, & F-K, 11
What is a subgrid model?

Express the coarse-grain averages of quantities (including the subgrid effects), e.g.:

\[ \frac{\partial \tau}{\partial t} \quad \frac{\partial u}{\partial x} \quad \frac{\partial u \tau}{\partial x} \]

As a function of the resolved coarse-grain fields

\[ \frac{\partial \tau}{\partial t} = \frac{\partial \tau}{\partial t} \quad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} \quad \frac{\partial u \tau}{\partial x} = \frac{\partial u \tau}{\partial x} + \frac{\partial u' \tau'}{\partial x} \]

Note that **nonlinear** terms require **special treatment**

And Couple different scales, small talks to large!
Different Uses, Different Needs

- **MOLES** (e.g., the CM2.4 movie before; grid 5–25km)
  - Mesoscale Ocean Large Eddy Simulation
  - Largest eddies are resolved—need smooth cutoff in mesoscale range

- **MORANS** (e.g., typical IPCC/CMIP models; grid >50km)
  - Mesoscale Ocean Reynolds-Averaged Navier-Stokes
  - Nothing resolved, unresolved to be parameterized

- **SMORANS** (e.g., Fox-Kemper et al., 2011; grid 1–10km)
  - Submesoscale Ocean...
  - Mesoscale resolved, submesoscale unresolved...

- **NOTE:** RANS contains all smaller scales that couple!
Extrapolate for historical perspective:
The Golden Era of Subgrid Modeling is Now!

<=SG Models=>

Ocean Model Resolution (km)

Euler Eqns.
Laplace Tidal Eqns.
Navier-Stokes Eqns.
Charney-von Neumann Numerical Weather Pred.
Smagorinsky Viscosity
Global Ocean DNS-

IPCC
Researchers have already cast much darkness on this subject and if they continue their investigations we shall soon know nothing at all about it.

--Mark Twain
The Character of the Mesoscale

- Boundary Currents
- Eddies
- $Ro = O(0.1)$
- $Ri = O(1000)$
- Full Depth
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly baroclinic & barotropic instability. Parameterizations of baroclinic instability (GM, Visbeck...).
A MOLES Closure: Smagorinsky & Kolmogorov vs. Leith & Kraichnan

Idea: Replace Eddy Momentum Fluxes with Artificially Inflated Viscosity

Relies on:
Energy Source, Dissipation, Flow & Dimensional Analysis
Truncation of Cascades

Power Spectrum:
Energy/Wavenumber

1941: Kolmogorov Envisions the Inertial Range

\[ \langle E \rangle = \frac{1}{V} \iiint \frac{1}{2} (u \cdot u) \, dV = \int_0^{\infty} E(k) \, dk. \]
1963: Smagorinsky Devises Viscosity Scaling, 
So that the Energy Flow is Preserved, 
but order-1 gridscale Reynolds #: \( Re^* = U L / \nu_* \)

\[
\nu_{*h} = \left( \frac{\Gamma_h \Delta x}{\pi} \right)^2 \sqrt{\left( \frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y} \right)^2 + \left( \frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x} \right)^2}.
\]
Except... Ocean Turbulence isn’t 3d Turbulence at the Gridscale

- The ocean is wide (10000km)
- But not deep (4km)
- So motions are largely 2d
- The layer of blue paint on a globe has roughly the right aspect ratio!
- MOLES grid aspect is similar
2d Turbulence Differs
(Kraichnan, 67)

2 Conserved Quantities: Energy and Enstrophy (vorticity variance)

Energy Cascades Upscale, Enstrophy Downscales...
2d Turbulence Differs

$E(k)$

$k^{-5/3}$

forcing

$\text{Re}^* = 1$

$k^{-3}$

$k_0$, $k_1$, $k_D$, $k$

$\frac{2\pi}{\Delta x}$

dissipation

1996: Leith Devises Viscosity Scaling, So that the Enstrophy Flow is Preserved

$\nu_* = \left( \frac{\Lambda \Delta x}{\pi} \right)^3 \nabla_h \left( \frac{\partial u_*}{\partial y} - \frac{\partial v_*}{\partial x} \right)$.
2-d Turbulence is different from Atmosphere (Ocean*?) macro-turbulence

* My student, Katie McCaffrey, is working on ocean spectra from obs.

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MOLES Turbulence Like Pot'l Enstrophy cascade, but divergent

2008: F-K & Menemenlis Revise Leith Viscosity Scaling, So that diverging, vorticity-free, modes are also damped

$$\nu_\ast = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla h q_2|^2 + \Lambda_d^6 |\nabla h (\nabla_h \cdot \mathbf{u}_\ast)|^2}.$$
Makes viscosity a bit bigger, especially near Eq.

Leith

F-K&M
But matters a lot for stability!

Figure 4. Maximum Courant number, \( w \Delta t / \Delta z \), for vertical advection. Gray line is from the LeithOnly integration and black line is from the LeithPlus integration.

Fox-Kemper & Menemenlis, 2008
It works here!
Even with irregular grid!
It works here!
Even with irregular grid!
It works here!
Even with irregular grid!
Spectra & Viscosity are good for MOLES, but... Asymptotics tell us to worry about scalar transport, not momentum for MORANS!

Equations for Large Scale Ocean Dynamics:

\[
(f_0 + \beta Y)\hat{z} \times \overline{u} = -\nabla_h \overline{p},
\]

\[
\partial_z \overline{p} = \overline{b},
\]

\[
\nabla_h \cdot \overline{u} + \partial_z \overline{w} = 0,
\]

\[
\partial_t \overline{b} + \overline{u} \cdot \nabla_h \cdot \overline{b} + \overline{w} \partial_z \overline{b} + \nabla_h \cdot \begin{pmatrix} \overline{u} \overline{b}' \end{pmatrix} + \partial_z \begin{pmatrix} \overline{w} \overline{b}' \end{pmatrix} = \kappa_u \partial_z^2 \overline{b}
\]

No more momentum fluxes!, i.e.,

Grooms, Julien, & F-K, 2011

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TESTING MORANS Closures: Validation & Spatial variations of Gent-McWilliams & Redi

Idea: Study the fluxes of passive tracers and reconstruct the flux-gradient relationship

Relies on: Unique Lagrangian Transport Operator for All Tracers
Mesoscale Eddy Parameterizations all have the form:

\[
\begin{bmatrix}
    u'\tau' \\
    v'\tau' \\
    w'\tau'
\end{bmatrix} = - \begin{bmatrix}
    M_{xx} & M_{xy} & M_{xz} \\
    M_{yx} & M_{yy} & M_{yz} \\
    M_{zx} & M_{zy} & M_{zz}
\end{bmatrix} \begin{bmatrix}
    \bar{\tau}x \\
    \bar{\tau}y \\
    \bar{\tau}z
\end{bmatrix}
\]

With John Dennis & Frank Bryan, we took a POP0.1° Normal-Year forced model (yrs 16-20)
Added 9 Passive tracers--restored x,y,z @ 3 rates
Kept all the eddy fluxes for passive & active
Coarse-grained to 2°, transient eddies, tracers to M

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Does this cover all the degrees of freedom?

More tracers does provide a just-determined or overdetermined (Moore-Penrose/least squares) problem for $M$ with a unique answer, but...

Different tracers will have different fluxes as they feel the subgrid ‘nooks and crannies’ of the mesoscale eddies!
\[ \mathbf{u}' \mathbf{\tau}' = -M \nabla \mathbf{\tau} \]

Sym Part = Anisotropic* Redi

\[
\begin{bmatrix}
\mathbf{u}' \mathbf{\tau}' \\
\mathbf{v}' \mathbf{\tau}' \\
\mathbf{w}' \mathbf{\tau}'
\end{bmatrix} = -
\begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{\nabla} z \cdot \mathbf{K} \cdot \hat{\nabla} z
\end{bmatrix}
\begin{bmatrix}
\tilde{\mathbf{\tau}}_x \\
\tilde{\mathbf{\tau}}_y \\
\tilde{\mathbf{\tau}}_z
\end{bmatrix}
\]

Antisym Part = Anisotropic* GM

\[
\begin{bmatrix}
\mathbf{u}' \mathbf{\tau}' \\
\mathbf{v}' \mathbf{\tau}' \\
\mathbf{w}' \mathbf{\tau}'
\end{bmatrix} = -
\begin{bmatrix}
0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z \\
\hat{x} \cdot \mathbf{K} \cdot \hat{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \hat{\nabla} z & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{\mathbf{\tau}}_x \\
\tilde{\mathbf{\tau}}_y \\
\tilde{\mathbf{\tau}}_z
\end{bmatrix}
\]

Yellow \(\mathbf{K}\) ‘are’ horizontal stirring & mixing

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Are Diffusivity Values Reasonable?

\[
\begin{bmatrix}
u' \tau' \\
v' \tau' \\
w' \tau'
\end{bmatrix} = -
\begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \nabla z \\
K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \nabla z \\
\hat{x} \cdot \mathbf{K} \cdot \nabla z & \hat{y} \cdot \mathbf{K} \cdot \nabla z & \nabla z \cdot \mathbf{K} \cdot \nabla z
\end{bmatrix}
\begin{bmatrix}
\bar{\tau}_x \\
\bar{\tau}_y \\
\bar{\tau}_z
\end{bmatrix}
\]

- Hor. Diffusivity is roughly \( \text{Trace}(\mathbf{M})/2 \)

- Peak of Diffusivity near \( 250 \text{ m}^2/\text{s} \)

- Median Diffusivity near \( 1000\text{m}^2/\text{s} \)

<6% negative
Could you have guessed it?
Result: Strong Anisotropy Along/Across Isopycnals

Mixing:

Stirring:
Result: \( \text{Redi } K = \text{GM } K \) (mostly)

If so these 2 components should match in Sym & Antisym.
Result:

\[ \text{Redi} \ K = GM \ K \text{(mostly)} \]

If so these 2 components should match in Sym & Antisym M
Result: Strong Anisotropy Along/Across PV Grads.

Mixing direction

Either along PV contours or across

1rst Eigenvector

2nd Eigenvector

Across PV contours

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Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations

**Fig. 1.** Annual mean thickness diffusivity ($K$) in m$^2$/s at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of $K$ are shown for the interior region only, i.e. values of $K$ in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear scale for the thickness diffusivity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The grid mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model's land mask.
But, how well does it work? Suppose we only plot values where different tracer sets agree...

Not so many trustworthy values!

Can't reject params!
Does this cover all the degrees of freedom?

More tracers does provide a just-determined or overdetermined (Moore-Penrose/least squares) problem for $M$ with a unique answer, but...

Different tracers will have different fluxes as they feel the subgrid ‘nooks and crannies’ of the mesoscale eddies!
\[ \frac{d\tau}{dt} = -\lambda(\tau - \tau_0) \]

\[ \overline{v' b'}_{\text{rec}} = -M \nabla \overline{b} \]

In idealized runs, can see the effect of restoring. Whatever we do, we need to get buoyancy right!

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In idealized setting, can do better

Reconstruction of eddy buoyancy fluxes

Using specially-tailored non-restored tracers improves estimate (error is now < 10%), but not feasible in realistic diagnosis.

In realistic diagnosis, we can improve the estimate a bit by approximating restoring effect
Sub-Mesoscale Parameterizations

Anyone who doesn't take truth seriously in small matters cannot be trusted in large ones either.

--Albert Einstein
The Character of the Submesoscale (Capet et al., 2008)

- Fronts
- Eddies
- $Ro = O(1)$
- $Ri = O(1)$
- Near-surface
- $1$–$10$ km, days


Parameterizations of baroclinic instability?
Mixed Layer Eddy Restratiﬁcation

Estimating eddy buoyancy/density ﬂuxes:

$$u'b' \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced overturning:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{z}$$

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Mixed Layer Eddy Restratiﬁcation

Estimating eddy buoyancy/density ﬂuxes:

\[ \overline{u'b'} \equiv \Psi \times \nabla \overline{b} \]

A submeso eddy-induced overturning:

\[ \Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \overline{b} \times \hat{z} \]

in ML only:

\[ \mu(z) = 0 \text{ if } z < -H \]
Mixed Layer Eddy Restratiﬁcation

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in ML only:
\[ \mu(z) = 0 \text{ if } z < -H \]

For a consistently restratiﬁng,
\[ \mathbf{w}' \mathbf{b}' \propto \frac{H^2}{|f|} |\nabla_{H} \bar{b}|^2 \]
Mixed Layer Eddy Restratiﬁcation

Estimating eddy buoyancy/density ﬂuxes:

\[ \mathbf{u}' \mathbf{b}' \equiv \Psi \times \nabla \bar{b} \]

A submeso eddy-induced overturning:

\[ \Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{z} \]

in ML only:

\[ \mu(z) = 0 \text{ if } z < -H \]

For a consistently restratiﬁng,

\[ \overline{w' \mathbf{b}'} \propto \frac{H^2}{|f|} |\nabla \bar{H} \bar{b}|^2 \]

and horizontally downgradient ﬂux.

\[ \overline{u' H \mathbf{b}'} \propto \frac{-H^2 \partial \bar{b}}{|f|} \nabla \bar{H} \bar{b} \]
Mixed Layer Eddy Restratiﬁcation

Estimating eddy buoyancy/density ﬂuxes:

\[ \mathbf{u}' \mathbf{b}' \equiv \nabla \bar{b} \times \nabla \bar{b} \]

A submeso eddy-induced overturning:

\[ \Psi = C \mu(z) \frac{H^2}{|f|} \nabla \bar{b} \times \mathbf{z} \]

in ML only:

\[ \mu(z) = 0 \text{ if } z < -H \]

For a consistently restratiﬁed:

\[ \mu(z) = 0 \text{ if } z < -H \]

and horizontally downgradient:

\[ \frac{\mathbf{u}' H \mathbf{b}'}{H^2} \propto \frac{\partial \bar{b}}{\partial z} \nabla_H \bar{b} \]

Estimating eddy buoyancy/density ﬂuxes:

Surface Temp.

200 m Temp.

Temp. x-z Section

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What does eddy restratification look like?

Parameterization Prediction
7d01h from 2d parameterization

Averaged MLE-resolving Model Solution
7d01h from 3d MITgcm (smoothed)

red = streamfunction
black = mean density
What does eddy restratification look like?

Time-Evolving Stratification ($N^2$)

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Mixed Layer Depth Bias Versus Observations (No MLE, Control)

Mixed Layer Depth Bias Versus Observations (With MLE)

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Physical Sensitivity of Ocean Climate to Submesoscale Eddy Restratification:

FFH implemented in CCSM (NCAR), CM2M & CM2G (GFDL)

NO RETUNING NEEDED!!!

Improves CFCs Passive tracer

Bias with MLE

Deep ML Bias reduced

From Fox-Kemper et al., 2011
Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC

Affects sea ice

NO RETUNING NEEDED!!!

These are impacts: bias change unknown

Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM+ minus CCSM-): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

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On a list of the 50 most important things to fix in the ocean model, Langmuir is number 51.

--Bill Large
The Character of the Langmuir Scale

- Near-surface
- Ro >> 1
- Ri < 1: Nonhydro
- 10-100m
- mins, hours
- w, u = O(20 cm/s)
- Stokes drift
- Eqtns: Craik-Leibovich
- unused params exist

Image: NPR.org, Deep Water Horizon Spill

Leibovich, 83
An Immature Improvement to Air-Sea BL

Mixing by Langmuir Turbulence
- Forced by wind and waves
  - i.e., Stokes drift & Eulerian Shear
- Scalings from LES, Observations disagree

We used a 2-part approximation
1) McWilliams & Sullivan (01) additional OBL mixing (within mixed layer)
2) Li & Garrett (98) Langmuir mixing depth (entrainment)

Roughly comparable to other schemes, but crude & incompletely validated
Needs only $u_*$, $u_s$ to work
Langmuir Mixing Forc    ed by Climatology

(Dong et al. Observations)

(Generalized Turbulent Langmuir)$^2$

Projection of $u^*$, $u_s$ into Langmuir Direction

\[
La_t^2 = \frac{|u^*|}{|u_s|} \left[ \frac{|u^*| + |u_s| \cos \theta}{|u_s| + |u^*| \cos \theta} \right]
\]
Tricky: Misaligned Wind & Waves

Van Roekel et al. 2011 (in prep)
Tricky: Misaligned Wind & Waves

Waves (Stokes Drift)

Wind

Van Roekel et al. 2011 (in prep)

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Tricky: Misaligned Wind & Waves

Van Roekel et al. 2011 (in prep)

Wind

Waves (Stokes Drift)

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Tricky: Misaligned Wind & Waves

Van Roekel et al. 2011 (in prep)
Coupling between Langmuir and Submeso? 

- Together?

- Separate?
The Game

Spin up a submeso-resolving, but not Langmuir resolving model

- 20kmx20kmx0.1km
- Grid 384x384x20
- 52m resolution

Interpolate down to Langmuir resolving LES

- 20kmx20kmx0.3km
- Grid 4096x4096x128
- 5m resolution

Run for 2 more days, then...

Day 6.5 of a Submeso Resolving run
Near Surf. Temp

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The Game

- Spin up a submeso-resolving, but not Langmuir resolving model
  - 20kmx20kmx0.1km
  - Grid 384x384x20
  - 52m resolution
- Interpolate down to Langmuir resolving LES
  - 20kmx20kmx0.3km
  - Grid 4096x4096x128
  - 5m resolution
- Run for 2 more days, then...

Day 6.5 of a Submeso Resolving run
Vert. Velocity

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Coupling Langmuir to Submesoscale?

Near-Surf Vert. Vel. With Stokes Drift

Near-Surf Temp. With Stokes Drift
Coupling Langmuir to Submesoscale?

Vertical Velocity
No Stokes Drift

Near-Surf. Temp.
No Stokes Drift

[Images of data plots showing vertical velocity and near-surface temperature]
Coupling Langmuir to Submesoscale?

From Scratch... No interpolation!

Near-Surf. Temp. No Stokes Drift


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Conclusions

Mesoscale, Submesoscale, and Langmuir scale phenomena all have a nontrivial affect on the global climate, thus need accurate parameterizations.

Parameterizations are developed by comparison to higher-resolution models, with careful diagnosis of interesting terms.

These high resolution models reveal primary balances and spatiotemporal dependence that should be approximated by the parameterizations.
Eden & Greatbatch (+others) propose that baroclinic instability's production of EKE from PE should guide M magnitude.
Locations of PE extraction are

Locations of large eigs of $K$
Result:
coarse KE $\rightarrow$ vertical structure of Mixing

$K \propto \sqrt{\langle KE \rangle}$

Even better with EKE!
Note--barotropic mode is in there!

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Comparisons with Marshall et al.

Abernathy et al. 09

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Comparisons with Marshall et al.

Abernathy et al 09

Critical Layer? thus nonlocal vert. modes?

Monday, June 13, 2011
Locations of PE extraction are

Locations of large eigs of $K$

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Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\overline{u^' \tau^'} \\
\overline{v^' \tau^'} \\
\overline{w^' \tau^'}
\end{bmatrix}
= -
\begin{bmatrix}
0 & 0 & -\hat{x} \cdot K \cdot \mathbf{\nabla} z \\
0 & 0 & -\hat{y} \cdot K \cdot \mathbf{\nabla} z \\
\hat{x} \cdot K \cdot \mathbf{\nabla} z & \hat{y} \cdot K \cdot \mathbf{\nabla} z & 0
\end{bmatrix}
\begin{bmatrix}
\overline{T_x} \\
\overline{T_y} \\
\overline{T_z}
\end{bmatrix}
\]

Atlantic Section

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Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\overline{u'\tau'} \\
\overline{v'\tau'} \\
\overline{w'\tau'}
\end{bmatrix} = -
\begin{bmatrix}
K_{xx} & K_{xy} & \hat{x}\cdot K\cdot \tilde{\nabla}z \\
K_{yx} & K_{yy} & \hat{y}\cdot K\cdot \tilde{\nabla}z \\
\hat{x}\cdot K\cdot \tilde{\nabla}z & \hat{y}\cdot K\cdot \tilde{\nabla}z & \tilde{\nabla}z\cdot K\cdot \tilde{\nabla}z
\end{bmatrix}
\begin{bmatrix}
\overline{\tau}_x \\
\overline{\tau}_y \\
\overline{\tau}_z
\end{bmatrix}
\]

Atlantic Section

Sym 3.1: Redi@lon=345E
Sym 3.2: Redi@lon=345E
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
    u' \tau' \\
    v' \tau' \\
    w' \tau'
\end{bmatrix}
= -
\begin{bmatrix}
    0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\
    0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\
    \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z & 0
\end{bmatrix}
\begin{bmatrix}
    \overline{T}_x \\
    \overline{T}_y \\
    \overline{T}_z
\end{bmatrix}
\]
Use a Natural, Mesoscale Eddy Environment to Test Out:

\[
\begin{bmatrix}
\overline{u' \tau'} \\
\overline{v' \tau'} \\
\overline{w' \tau'}
\end{bmatrix}
= - \begin{bmatrix}
K_{xx} & K_{xy} & \hat{x} \cdot K \cdot \hat{\nabla} z \\
K_{yx} & K_{yy} & \hat{y} \cdot K \cdot \hat{\nabla} z \\
\hat{x} \cdot K \cdot \hat{\nabla} z & \hat{y} \cdot K \cdot \hat{\nabla} z & \hat{\nabla} z \cdot K \cdot \hat{\nabla} z
\end{bmatrix}
\begin{bmatrix}
\overline{\tau_x} \\
\overline{\tau_y} \\
\overline{\tau_z}
\end{bmatrix}
\]
Scaling: Antisymmetric part

\[ \psi \]

\[ \frac{Ri^{-3/2} M^2 H^2}{|f|} \]
Scaling: Larger symmetric eigenvalue

\[ \lambda_1 \]

\[ \left( \frac{1}{400} \right) Ri^{-1/2} N^2 H^2 \frac{\left| f \right|}{|f|} \]
Scaling: Smaller symmetric eigenvalue

\[ \lambda_2 \]

\[ \left( \frac{1}{6} \right) \frac{Ri^{-2} M^4 H^2}{N^2 |f|} \]
Overdetermined vs. Underdetermined:

Mean values across eddying region

\[ A_{12} \text{ and } \psi_{sym} \text{ are mathematically equivalent!} \]
Parameterization of Finite Amp. Eddies: Ingredients
Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

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Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

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Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

Eddy Velocity Saturates

- basin-avg. pert. KE
- linear predict. pert. KE.
- initial mean KE: $\frac{1}{2}(M^2 H/f)^2$
- avg. pert. $\sqrt{v^2}$ in front
Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

Eddy Velocity Saturates Near Mean KE

\[ \frac{N^2}{f^2} \]

\[ \text{time (days)} \]

\[ \text{kinetic energy (m}^2\text{/s}^2) \]

\[ \text{initial mean KE}^2: \frac{1}{2}(M^2 H/f)^2 \]

\[ \text{avg. pert. } \vec{v}^2 \text{ in front} \]

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Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

Vert. Excursions $(b_{rms}/N^2)$ scale with $H$

Eddy Velocity Saturates

Near Mean KE

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Finite Amplitude

Vert. Excursions \( \frac{b_{\text{rms}}}{N^2} \) scale with \( H \)

Eddy Velocity Saturates Near Mean KE

Eddy Fluxes are at nearly 1/2 the mean isopycnal slope

\[ \text{initial mean KE}^2: \frac{1}{2}(M^2 H/f)^2 \]

\[ \text{avg. pert. } \sqrt{v^2} \text{ in front} \]
Parameterization of Finite Amp. Eddies: Ingredients

Finite Amplitude

Scale with $H$

Linear Solution $\langle w'b' \rangle$

for vert. structure.

As in Branscome '83...

$\left( \frac{b'_{rms}}{N^2} \right)$

scale with $H$

Eddy Velocity Saturates

Near Mean KE

Eddy Fluxes

are at nearly

1/2 the mean

isopycnal slope

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