Surface Waves in Turbulent and Laminar Submesoscale Flow

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The Earth’s Climate System is driven by the Sun’s light (minus outgoing infrared) on a global scale.

Dissipation concludes turbulent cascades on scales about a trillion times smaller.

Fig. 1. The global annual mean Earth’s energy budget for the Mar 2000 to May 2004 period (W m⁻²). The broad arrows indicate the schematic flow of energy in proportion to their importance.

Fig. 1. TOA annualized ERBE zonal mean net radiation (W m⁻²) for Feb 1985–Apr 1989.

Forcing range
Inertial range
Dissipation range

E(k)

1/L
K_F
K_D

Polar
Deficit

Eqtr Excess

Polar
Deficit

Poleward energy Transfer needed by Atmosphere & Ocean
Air-Sea Flux Errors vs. Data

Heat capacity & mode of transport is different in A vs. O

Resolution will be an issue for centuries to come!

IPCC: Intergovernmental Panel on Climate Change

They won the Nobel (Peace) Prize with Al Gore

Here are the collection of IPCC models...

If we can’t resolve a process, we need to develop a parameterization or subgrid model of its effect.
What is a parameterization/subgrid model?

Fluid equations for A&O are PDEs (Rotating, Stratified Navier-Stokes), but we cannot resolve to dissipation, so we use statistical or bulk subgrid models to capture multiscale interactions:

Express the coarse-grain averages of quantities (including the subgrid effects), e.g.:

\[
\frac{\partial \tau}{\partial t} + \frac{\partial \bar{u}}{\partial x} \frac{\partial \bar{u}}{\partial x} + \frac{\partial u \tau}{\partial x}
\]

As a function of the resolved coarse-grain fields

\[
\frac{\partial \tau}{\partial t} = \frac{\partial \bar{\tau}}{\partial t} \quad \frac{\partial \bar{u}}{\partial x} = \frac{\partial \bar{u}}{\partial x} \quad \frac{\partial u \tau}{\partial x} = \frac{\partial u \bar{\tau}}{\partial x} + \frac{\partial u' \tau'}{\partial x}
\]

Note that nonlinear terms require special treatment

These couple different scales, small talks to large
The Ocean is Vast & Diverse: Q: What processes to parameterize? 
Today’s A: Unresolved Upper Ocean with Air–Sea Impact

CCSM4 resolves
Fundamental Equations of Motion of a Fluid

The following constitutes, in principle, a complete set of equations for an inviscid fluid heated at a rate $\dot{Q}$ and whose composition, $S$, changes at a rate $\dot{S}$.

\[
\frac{D?}{Dt} \equiv \frac{\partial?}{\partial t} + \mathbf{v} \cdot \nabla? \quad (\text{F.1})
\]

Evolution equations for velocity, density and composition:

\[
\frac{D\mathbf{v}}{Dt} = -\nabla \frac{p}{\rho} + \mathbf{F}', \quad \frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0, \quad \frac{DS}{Dt} = \dot{S}.
\]

Internal energy equation or entropy equation:

\[
\frac{DI}{Dt} - \frac{p}{\rho} \nabla \cdot \mathbf{v} = \dot{Q}_T, \quad \frac{D\eta}{Dt} = \frac{1}{T} \dot{Q}. \quad (\text{F.2})
\]

where $\dot{Q}_T = \dot{Q} + \mu \dot{S}$ is the total rate of energy input.

Fundamental equation of state:

\[
I = I(\rho, S, \eta). \quad (\text{F.3})
\]

Diagnostic equations for temperature and pressure:

\[
T = \left( \frac{\partial I}{\partial \eta} \right)_{\alpha, S}, \quad p = -\left( \frac{\partial I}{\partial \alpha} \right)_{\eta, S}. \quad (\text{F.4})
\]

Vallis, 06

9 Variables 9 Equations. Brutal but complete.
With nearly incompressible (small density variations) approximation & approximated rotating Earth: A simpler set of 5 vars

Summary of Boussinesq Equations

The simple Boussinesq equations are, for an inviscid fluid:

- **Momentum equations:**
  \[
  \frac{D\mathbf{u}}{Dt} + f \times \mathbf{u} = -\nabla \phi + b \mathbf{k}, \quad (B.1)
  \]

- **Mass conservation:**
  \[
  \nabla \cdot \mathbf{u} = 0, \quad (B.2)
  \]

- **Buoyancy equation:**
  \[
  \frac{Db}{Dt} = \dot{b}. \quad (B.3)
  \]

Vallis, 06

If you want, it's easy to distinguish buoyancy into contributions from Temperature and from Salinity
Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:
Inviscid \((Re \gg 1)\), rapidly rotating \((Ro \ll 1)\), and thin* \((L \gg H)\)

**Full Momentum**

\[
\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla \phi + b \mathbf{k} + \nu \nabla^2 \mathbf{v}
\]

\[
Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri \equiv \frac{\partial b}{\partial z} \quad \alpha = \frac{H}{L}
\]

*closey related to strong statification & ocean dimensions
Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:
Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Horizontal) Geostrophic Balance

\[
\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla \phi + bk + \nu \nabla^2 \mathbf{v}
\]

\[
Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri \equiv \frac{\partial b}{\partial z} \quad \alpha = \frac{H}{L}
\]

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Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:
Inviscid ($Re \gg 1$), rapidly rotating ($Ro \ll 1$), and thin* ($L \gg H$)

(Vertical) Hydrostatic Balance

\[
\frac{Dv}{Dt} + f \times v = -\nabla \phi + bk + \nu \nabla^2 v
\]

Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri \equiv \frac{\partial b}{\partial z} \quad \alpha = \frac{H}{L}

* closely related to strong stratification & ocean dimensions
Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit: Inviscid (Re$\gg$1), rapidly rotating (Ro$\ll$1), and thin* (L$\gg$H)

(Combined) Thermal Wind Balance

\[ \mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b \]

Taken together with the forcing (air-sea) of buoyancy and the advection of buoyancy by this flow--you have the tools to study large-scale ocean physics!
Let's see some examples of Bousinesq, Hydrostatic Models at work in the mesoscale (10-100km) & submesoscale (100m-10km)
Big, Deep (mesoscale) interact with Little, Shallow (submeso)

The Character of the Submesoscale

(Capet et al., 2008)

- Fronts
- Eddies
- $Ro=O(1)$
- $Ri=O(1)$
- near-surface
- 1-10 km, days

Eddy processes often baroclinic instability

Parameterizations of submesoscale baroclinic instability?


Physical Sensitivity of Ocean Climate to MLE: (submeso) Mixed Layer Eddy Restratification Improves CFCs (water masses)

**Error w/o MLE**


A consistently restratifying, and horizontally downgradient flux.

\[ \overline{w'b'} \propto \frac{H^2}{|f|} \left| \nabla_H \overline{b} \right|^2 \]

\[ \overline{u'Hb'} \propto \frac{-H^2 \frac{\partial \overline{b}}{\partial z}}{|f|} \nabla_H \overline{b} \]
So, we’ve seen that we can study a small-scale system (1-10km submeso mixed layer eddies), derive parameterizations, and then use them to improve climate models & assess impact globally.

This particular process relied heavily on thermal wind scaling relationships.

But, what about the effects of things that aren’t geostrophic & hydrostatic?

For example, waves and near-surface 3d turbulence.
Surface Wave Primer

Look for fast, small solutions of the Boussinesq Equations:

The irrotational, incompressible flow obeys

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

The boundary conditions are:

- **Solid Bottom**
  $$w = \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -H$$

- **Pressure Matching (dynamic)**
  $$p = 0 \quad \text{at} \quad z = \eta$$

- **Velocity Matching (kinematic)**
  $$\frac{D\eta}{Dt} = w_\eta \quad \text{at} \quad z = \eta$$
Surface Wave Primer

Look for fast, small solutions of the Boussinesq Equations:

Linearized for not steep waves

The irrotational, incompressible flow obeys

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

The boundary conditions are (small steepness):

<table>
<thead>
<tr>
<th>Solid Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w = \frac{\partial \phi}{\partial z} = 0$ at $z = -H$</td>
</tr>
</tbody>
</table>

Pressure Matching (dynamic)

| $\frac{\partial \phi}{\partial t} = -g \eta$ at $z = 0$ |

Velocity Matching (kinematic)

| $\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z}$ at $z = 0$ |
Particle motions

The $u$, $v$, decay exponentially toward the bottom with decay scale proportional to the wavelength.

Thus, $kH$ is a measure of depth

$k = \frac{2\pi}{\text{wavelength}}$

$a = \text{amplitude}$

\[\omega = \sqrt{gk}\]
\[c_p = 2c_g = \sqrt{g/k}\]

Deep water waves don’t “feel” the bottom. Implies nonhydrostatic (\(Ro \gg 1\)) & fast timescale.
The Character of the Langmuir Scale

- Near-surface
- $Ro \gg 1$
- $Ri < 1$: Nonhydro
- 1-10m
- 10s to mins
- $w, u = O(10\text{cm/s})$
- Stokes drift
- Eqtns: Craik-Leibovich
- Params: McWilliams & Sullivan, 2000, etc.

Image: NPR.org, Deep Water Horizon Spill

Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).
Craik–Leibovich Boussinesq

Old Boussinesq (written in vortex force form)
\[
\frac{\partial \mathbf{v}}{\partial t} + \left[ \mathbf{f} + \nabla \times \mathbf{v} \right] \times \mathbf{v} = -\nabla \pi + b k + \nu \nabla^2 \mathbf{v}
\]

\[
\frac{\partial b}{\partial t} + \mathbf{v} \cdot \nabla b = 0 \quad \nabla \cdot \mathbf{v} = 0
\]

Craik–Leibovich Boussinesq
\[
\frac{\partial \mathbf{v}}{\partial t} + \left[ \mathbf{f} + \nabla \times \mathbf{v} \right] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^{\dagger} + b k + \nu \nabla^2 \mathbf{v}
\]

\[
\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0
\]

\[
\nabla \cdot \mathbf{v} = 0
\]

\[
\mathbf{v}_s = \text{Stokes Drift}
\]
What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

\[ u^L(x_p(t_0), t) - u^E(x_p(t_0), t) \approx [x_p(t) - x_p(t_0)] \cdot \nabla u^E(x_p(t_0), t) \]

\[ \approx \left[ \int_{t_0}^{t} u^E(x_p(t_0), s') ds' \right] \cdot \nabla u^E(x_p(t_0), t). \]

Examples:

Monochromatic:

\[ u^S = \hat{e}^w \frac{8\pi^3 a^2 f_p^3}{g} e^{-\frac{8\pi^2 f^2}{g}} = \hat{e}^w a^2 \sqrt{gk^3} e^{2kz}. \]

Spectrum:

\[ u^S = \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^{\pi} (\cos \theta, \sin \theta, 0) f^3 S_{f\theta}(f, \theta) e^{-\frac{8\pi^2 f^2}{g}} d\theta df. \]
How well do we know Stokes Drift? <50% discrepancy

RMS error in measures of surface Stokes drift, 2 wave models (left), model vs. altimeter (right)

Year 2000 data & models

Craik–Leibovich Boussinesq

Formally a multiscale asymptotic equation set:

3 classes: Small, Fast; Large, Fast; Large, Slow

Solve first 2 types of motion in the case of limited slope (ka), irrotational → Deep Water Waves!

Average over deep water waves in space & time,

\[
\begin{align*}
\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) &= -\nabla \pi^\dagger + bk + \nu \nabla^2 \mathbf{v} \\
\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b &= 0 \quad \nabla \cdot \mathbf{v} = 0
\end{align*}
\]

\[\mathbf{v}_s = \text{Stokes Drift}\]

Craik & Leibovich 1976; Gjaja & Holm 1996; McWilliams et al. 2004
CLB as equations for Large Eddy Simulations: Tricky: Misaligned Wind & Waves

Tricky: Misaligned Wind & Waves

Tricky: Misaligned Wind & Waves

Tricky: Misaligned Wind & Waves

Generalized Turbulent Langmuir No.,
Projection of $u^*$, $u_s$ into Langmuir Direction

A scaling for LC strength & direction!

Why? Vortex Tilting Mechanism

In CLB: Tilting occurs in direction of $u_L = v + v_s$

Misalignment enhances degree of wave-driven LT

Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).
Recall our problem with the (submeso) Mixed Layer Eddy Restratification—Southern Ocean too shallow!

Sallee et al. (2013) have shown that a too shallow S. Ocean MLD is true of most* present climate models


*true for CMIP5 multi-model ensemble
Data + LES, Southern Ocean mixing energy: Langmuir (Stokes-drift-driven) and Convective

So, waves can drive mixing via Stokes drift (combines with cooling & winds)

Including Wave-driven Mixing (Harcourt 2013 parameterization) Deepens the Mixed Layer!

What about Langmuir-Submeso Interactions?

Perform large eddy simulations (LES) of Langmuir turbulence with a submesoscale temperature front.

Use NCAR LES model to solve Craik-Leibovich equations (Moeng, 1984, McWilliams et al, 1997).

\[ \frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \text{SGS} \]

\[ \nabla \cdot \mathbf{u} = 0 \]

\[ \frac{\partial \mathbf{u}}{\partial t} + (\omega + f \hat{z}) \times \mathbf{u}_L = -\nabla \pi - \frac{g \rho \hat{z}}{\rho_0} + \text{SGS} \]

Computational parameters:
Domain size: 20km x 20km x -160m
Grid points: 4096 x 4096 x 128
Resolution: 5m x 5m x -1.25m
Overall results

- Strong interactions between small & large scales are rare in this configuration.

- Two relatively independent turbulent spectral cascades near the surface. Only one (submeso) at depth.

- Presence of waves greatly changes small scale instability character from symmetric instability to gravitational---this will matter!

Zoom: Submeso-Langmuir Interaction!

What’s plotted are surfaces of large vert. velocity, colored by temperature.
Diverse types of interaction

Slide & Movies by Peter Hamlington

So, Waves can Drive turbulence that affect larger scales indirectly:

What about direct effects of waves on larger scales?

Recall, from regular Boussinesq Equations:

\[(\text{Combined)} \text{ Thermal Wind Balance}\]

\[\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b\]
So, Waves can Drive turbulence that affect larger scales indirectly:

What about direct effects of waves on larger scales?

Now, Craik-Leibovich Boussinesq Equivalent:

(Combined) Lagrangian Thermal Wind Balance

\[ f \times \frac{\partial}{\partial z} (v + v_s) = f \times \frac{\partial v_L}{\partial z} = -\nabla b \]

Now the temperature gradients govern the Lagrangian flow, not the Eulerian!

So, can we just forget the whole thing and interpret large scales as Lagrangian velocities?

\[
[\mathbf{f} + \nabla \times \mathbf{v}] \times \frac{\partial}{\partial z} (\mathbf{v} + \mathbf{v}_s) = -\nabla b
\]

Not quite, because Ro > 0 corrections are different!

The “Ro” for waves, is big *more often* than Ro is, especially for wide, shallow currents in a mixed layer.

**Figure 1.** Estimated ratio \(\epsilon/R\) \(\approx (|\mathbf{u}_s \cdot \mathbf{u}| h_{14}) / (|\mathbf{u}|^2 h_{14})\) governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity (\(\mathbf{u}\)) is taken as the AVISO weekly satellite geostrophic velocity or –\(\mathbf{u}_s\) (for anti-Stokes flow) if \(|\mathbf{u}_s| > |\mathbf{u}|\). The front/filament depth (\(h\)) is estimated as the mixed layer depth from the de Boyer Montégut et al. (2004) climatology. An exponential fit to the Stokes drift of the upper 9m projected onto the AVISO geostrophic velocity provides \(\mathbf{u}_s \cdot \mathbf{u}\) and \(h_{14}\). Stokes drift is taken from the WaveWatch-3 simulation described in Webb & Fox-Kemper (2011). \(\mathbf{u}_s\), and \(h\) are all for the year 2000, while \(h\) is from a climatology of observations over 1961-2008. The year 2000 average of \(\epsilon/R\) is shown.

Waves (Stokes Drift Vortex Force) -> Submeso, Meso: An example

Initial Submeso Front
Contours: 0.1

Perturbation on that scale due to waves
Contours: 0.014

So, no problems?

Just crunch away with CLB?

Let’s revisit our assumptions for scale separation:

- CLB wave equations require limited *wave steepness* and irrotational flow
- Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...

Steep waves break→vortex motion & small scale turbulence!

Power Spectrum of wave height

\[
\langle \eta^2 \rangle = \int_0^\infty E(k) dk = C_0 + \int_{k_n}^\infty C_1 k^{-2} dk
\]

Power Spectrum of wave steepness: INFINITE!

\[
\langle k^2 \eta^2 \rangle = \int_0^\infty k^2 E(k) dk = D_0 + \int_{k_n}^\infty D_1 dk
\]
Conclusions

Climate modeling is challenging partly due to the vast and diverse scales of fluid motions.

In the upper ocean, horizontal scales as big as basins, and as small as meters contribute non-negligibly to the air-sea exchange.

Process models, especially those spanning a whole or multiple scales, are needed to study these connections and improve subgrid models.

Interesting are the submeso to Langmuir scales, as nonhydro. & ageostrophic effects begin to dominate.

The CLB are good for LES & analysis in this range, but cannot capture some effects of small, steep waves (breaking, spray, nearshore, etc.)
Extrapolate for historical perspective: The Golden Era of Subgrid Modeling is Now!

All papers at: fox-kemper.com/research
Mixed Layer Eddy Restratification

Estimating eddy buoyancy/heat flux:
\[
\mathbf{u}' \mathbf{b}' \equiv \Psi \times \nabla \bar{b}
\]

A submeso eddy-induced streamfunction:
\[
\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \nabla \bar{b}
\]

in ML only:
\[
\mu(z) = 0 \text{ if } z < -H
\]

For a consistently restratifying, the buoyancy flux is:
\[
\mathbf{w}' \mathbf{b}' \propto \frac{H^2}{|f|} \left| \nabla H \bar{b} \right|^2
\]

and horizontally downgradient flux:
\[
\mathbf{u}' H \mathbf{b}' \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla H \bar{b}
\]

Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC

Affects sea ice

NO RETUNING NEEDED!!!

These are impacts: bias change unknown