The Importance of Scale-Aware Physical Parameterizations in Mesoscale to Submesoscale Permitting Simulations

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Idea: Finally, computers are fast enough that we can resolve eddies in fully coupled climate simulations, but our subgrid models are suspect or inappropriate. What to do?
The Earth’s Climate System is forced by the Sun on a global scale (20,000km pole to pole).

Next-gen. ocean climate models simulate globe to 10km: Mesoscale Ocean Large Eddy Simulations (MOLES).

Turbulence cascades to scales about 10 billion times smaller.

Transmitting a single variable from ocean DNS: 325,000 (a year of the internet).
Key Concept for Mesoscale Ocean Large Eddy Simulations (MOLES): Gridscale Nondimensional Parameters

Gridscale Reynolds:\(^1\):

\[ Re^* = \frac{U^* \Delta x}{\nu^*} \]

Gridscale Péclet:\(^1\):

\[ Pe^* = \frac{U^* \Delta x}{\kappa^*} \]

Gridscale Rossby:

\[ Ro^* = \frac{U^*}{f \Delta x} \]

Gridscale Richardson:

\[ Ri^* = \frac{\Delta b^*^2 \Delta z}{\Delta U^*^2} \]

Gridscale Burger:

\[ Bu^* = \frac{N^*^2 \Delta z^2}{f^2 \Delta x^2} \sim Ro^*^2 Ri^* \]

Asterisks denote *resolved* quantities, rather than true values

\(^1\) Gridscale Reynolds and Péclet numbers MUST be O(1) for numerical stability

A Global Parameterization of Mixed Layer Eddy Flow & Scale Aware Restratification validated against simulations


\[
\mathbf{u}' \mathbf{b}' \equiv \Psi \times \nabla \overline{\mathbf{b}}
\]

\[
\Psi = \begin{bmatrix} \Delta x \\ \frac{L_f}{f} \end{bmatrix} \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \overline{\mathbf{b}} \times \hat{\mathbf{z}}
\]

Compare to the original singular, unrescaled version

\[
\Psi = \begin{bmatrix} C_e H^2 \mu(z) \\ |f| \end{bmatrix} \nabla \overline{\mathbf{b}} \times \hat{\mathbf{z}}
\]

New version handles the equator, and averages over many fronts
Purely advective, gets correct rate of restratification (akin to Gent-McWilliams 90).

In adiabatic situations, should pair nicely with an isopycnal diffusivity of equal magnitude (akin to Redi, see Bachman & F-K 13).

In diabatic situations (it is the mixed layer), may need diabatic parameterization (Brüggemann & Eden).

Tested in global 10km and finer regional simulations, turns itself off fairly appropriately in submeso-permitting regime.

What about the mesoscale-permitting?

What do we do when some mesoscale eddies are resolved and some are not?

This regime is fairly compared to Large Eddy Simulations, typical of much smaller scales.

How do we adapt the technology of LES to these scales?

Should we just turn off GM to allow partially-resolved eddies to get all APE (e.g., Hallberg, 2013)?

3D Turbulence Cascade

Smagorinsky (1963) Scale & Flow Aware Viscosity Scaling, So the Energy Cascade is Preserved, and $Re^* = \frac{U^* \Delta x}{\nu^*} = O(1)$

$$v_{*h} = \left(\frac{\gamma h \Delta x}{\pi}\right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y}\right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x}\right)^2}$$
**2D Turbulence Differs**

Leith (1996) Devises Viscosity Scaling, So that the Enstrophy Cascade is preserved, and \( \text{Re}^* = \frac{U^* \Delta x}{\nu^*} = O(1) \)

\[
\nu_{2d}^* = \left( \frac{\Lambda_{2d} \Delta x}{\pi} \right)^3 \left| \nabla_h q_{2d}^* \right|
\]

\[q_{2d}^* = \frac{\partial u^*}{\partial y} - \frac{\partial v^*}{\partial x}\]
Leith-Plus Parameterization in the high-resolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions without retuning:

1/12 degree Fram Strait, Temperature at 263m 1/48 degree

F-K & Menemenlis (08): Revise Leith viscosity to quasi-2d, by damping diverging, vorticity-free, modes, too.
Leith-Plus Parameterization in the high-resolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions without retuning:

\[
v_\ast = \left( \frac{\Delta x}{\pi} \right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_{d}^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_{\ast})|^2}
\]

Lab. Sea, Temperature at 263m
2d (SWE) test of MOLES Subgrid models

Pietarila Graham & Ringler, 2013

Harmonic/Biharmonic/Numerical
- Many. Often not scale- or flow-aware
- Griffies & Hallberg, 2000, is one aware example
- Chen, Q., Gunzburger, M., Ringler, T., 2011
  - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
  - Approximate Deconvolution Method
  - Stochastic & Statistical Parameterizations

In the Graham & Ringler comparison, Leith wins!
- over tuned harmonic, tuned biharmonic, Smagorinsky, LANS-alpha, & Anticipated PV

Fig. 21. Enstrophy spectra for benchmark (solid black), hyper-viscous $8192^2$ Truth
1008$^2$ LES in color

Leith wins!
Is 2D Turbulence a good proxy for neutral flow?

For a few eddy time-scales QG & 2D AGREE (Bracco et al. '04)

Barotropic Flow & Stratified Turbulence ($Ro \gg 1$, $Ri \gg 1$) are 2d analogs

Bolus Fluxes-- Divergent 2d flow

Sloped, not horiz.

Surface Effects?

Nurser & Marshall, 1991 JPO
QG Leith:

\[ \nabla_h^2 \psi^* = q_{2d}^* \]

\[ q_{ag}^* = \beta y + \nabla_h^2 \psi^* + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi^*}{\partial z} \right) \]
QG Turbulence: Pot’l Enstrophy cascade (potential vorticity\(^2\))

Consistent with QG only if scaling applies to ALL Pot’l Enstrophy sinks—Viscosity, Diffusivity, AND GM Coefficient:

\[
\nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg} \Delta x}{\pi}\right)^3 |\nabla q_{qg}|.
\]

\[
Re^* = \frac{U^* \Delta x}{\nu^*}
\]

\[
Pe^* = \frac{U^* \Delta x}{\kappa^*}
\]
And QG pot’l enstrophy Leith is ... now working in MITgcm

Scott Bachman (DAMTP) has implemented this QG Leith closure in the MITgcm

Both Germano Dynamic and Fixed Coefficient

\[ \Lambda_{qg} = \Lambda_{qg}(x, y, z, t) \quad \Lambda_{qg} = 1 \]

\[ \nu_{qg} = \left( \frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}| = \left( \frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 \left| \nabla_h \left[ \beta y + \nabla_h^2 \psi + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) \right] \right|. \]

\[ \nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left( \frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}|. \]
This Slide & Movies: S. Bachman

We’ll test this in a channel model, using three different resolutions:

The fastest growing mode is better resolved the higher the resolution, so the spindown will be slower for the coarser runs.

But the QG dissipation / diffusivity scheme is able to compensate!

Old Method=Smagorinsky viscosity with only implicit numerical diffusivity, no GM

New Method=QG Leith

\[ dx = \frac{L_d}{4} \]

\[ dx = \frac{L_d}{2} \]

\[ dx = L_d \]
Comparing 9 different subgrid closures with $dx = 0.4 \ L_d$
Comparing 9 different subgrid closures with $dx=0.2\ L_d$
Comparing 9 different subgrid closures with $dx=2 \ L_d$
But...we need to be careful of when QG isn’t appropriate:

- Stretching term can be too large when unstratified—use gridscale Burger number to determine when:
  \[
  Bu^* = \frac{N^*}{f} \frac{\Delta z^2}{\Delta x^2} \sim Ro^* \frac{u^*}{Ri^*}
  \]

- \( \frac{\nu_{qg}^*}{\nu_{2d}^*} \approx \frac{\nabla h q_{qg}^*}{\nabla h q_{2d}^*} \sim 1 + Bu^* \sim 1 + Ro^* \frac{u^*}{Ri^*} \)

- Surface QG has different spectral characteristics—we have a theory, but simultaneous implementation unclear
Conclusions

Promising mesoscale method: Realistic tests next!

- QG Leith=viscosity, Redi diffusivity, *and* GM transfer coeff.

- Ensures $O(1)$ gridscale Reynolds & Péclet

- Revert to 2D Leith when QG is inappropriate

- QG only if gridscale Burger near 1, gridscale Richardson $>1$

Our results suggest QG Leith will deliver the proven plug&play capability of LeithPlus with improved QG-based physics—Will matter most where stretching terms or APE balance are important, e.g., WBC.

Since it was built to keep gridscale Re and Pe $O(1)$, we expect to have spurious diapycnal mixing small (Ilicak et al., 2012). Testing now, based on buoyancy classes and passive tracers.