Ready to Resolve: Subgrid Parameterization for Tomorrow's Climate Models

Idea: Finally, computers are fast enough that we can resolve eddies in fully coupled climate simulations, but our subgrid models are suspect or inappropriate. What to do?
The Earth’s Climate System is forced by the Sun on a global scale (24,000km)

Next-gen. ocean climate models simulate globe to 10km: Mesoscale Ocean Large Eddy Simulations (MOLES)

Turbulence cascades to scales about 10 billion times smaller

All <10km is parameterized
Key Concept for Mesoscale Ocean Large Eddy Simulations (MOLES): Gridscale Nondimensional Parameters

Gridscale Reynolds:\n\[ Re^* = \frac{U^* \Delta x}{\nu^*} \]

Gridscale Péclet:\n\[ Pe^* = \frac{U^* \Delta x}{\kappa^*} \]

Gridscale Rossby:\n\[ Ro^* = \frac{U^*}{f \Delta x} \]

Gridscale Richardson:\n\[ Ri^* = \frac{\Delta b^* \Delta z}{\Delta U^*} \]

Gridscale Burger:\n\[ Bu^* = \frac{N^*^2 \Delta z^2}{f^2 \Delta x^2} \sim Ro^*^2 Ri^* \]

Asterisks denote *resolved* quantities, rather than true values

\(^1\) Gridscale Reynolds and Péclet numbers MUST be O(1) for numerical stability

Smagorinsky (1963) Scale & Flow Aware Viscosity Scaling, So the Energy Cascade is Preserved, and $Re^* = \frac{U^* \Delta x}{\nu^*} = O(1)$

$$v_{*h} = \left( \frac{\gamma_h \Delta x}{\pi} \right)^2 \sqrt{\left( \frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y} \right)^2 + \left( \frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x} \right)^2}$$
2D Turbulence Differs

R. Kraichnan, 1967 JFM

Leith (1996) Devises Viscosity Scaling, So that the Enstrophy Cascade is preserved, and $Re^* = \frac{U^* \Delta x}{\nu^*} = O(1)$

$$\nu^*_{2d} = \left( \frac{\Lambda_{2d} \Delta x}{\pi} \right)^3 |\nabla_h q^*_{2d}|$$

$$q^*_{2d} = \frac{\partial u^*}{\partial y} - \frac{\partial v^*}{\partial x}$$
Leith-Plus Parameterization in the high-resolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions without retuning:

1/12 degree Fram Strait, Temperature at 263m 1/48 degree

F-K & Menemenlis (08): Revise Leith viscosity to quasi-2d, by damping diverging, vorticity-free, modes, too.

\[
v_* = \left( \frac{\Delta x}{\pi} \right)^3 \sqrt{\Lambda^6 |\nabla_h g_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}
\]

Leith Plus
Leith-Plus Parameterization in the high-resolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions without retuning:

\[
v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h g_{22}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot u_*)|^2}
\]

Lab. Sea, Temperature at 263m
2d (SWE) test of MOLES Subgrid models

Pietarila Graham & Ringler, 2013

Harmonic/Biharmonic/Numerical
- Many. Often not scale- or flow-aware
- Griffies & Hallberg, 2000, is one aware example
- Chen, Q., Gunzburger, M., Ringler, T., 2011
  - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
  - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations

Fig. 21. Enstrophy spectra for benchmark (solid black), hyper-viscous

In the Graham & Ringler comparison, Leith wins!
over tuned harmonic, tuned biharmonic, Smagorinsky,
LANS-alpha, & Anticipated PV
Is 2D Turbulence a good proxy for neutral flow?

For a few eddy time-scales QG & 2D AGREE (Bracco et al. ’04)

Barotropic Flow & Stratified Turbulence (Ro>>1, Ri>>1) are 2d analogs

Bolus Fluxes-- Divergent 2d flow

Sloped, not horiz.

Surface Effects?
QG Turbulence: Pot’l Enstrophy cascade
(potential vorticity$^2$) J. Charney, 1971 JAS

\[ E(k) \]

Spectral Density of Kinetic Energy

Inverse Energy Cascade

Potential Enstrophy Cascade

Forcing

\[ \mathcal{E} \]

\[ \eta \]

\[ \frac{2\pi}{\Delta x} \]

\[ k_0 \]

\[ k \]

\[ k_1 \]

\[ k_D \]

QG Leith:

\[ \nabla_h^2 \psi^* = q_{2d}^* \]

\[ q_{ag}^* = \beta y + \nabla_h^2 \psi^* + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi^*}{\partial z} \right) \]
Consistent with QG only if scaling applies to ALL Pot’l Enstrophy sinks—Viscosity, Diffusivity, AND GM Coefficient:

\[ \nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left( \frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{g}|. \]
And QG pot’l enstrophy Leith is ... now working in MITgcm

Scott Bachman (DAMTP) has implemented this QG Leith closure in the MITgcm

Both Germano Dynamic and Fixed Coefficient

\[ \Lambda_{qg} = \Lambda_{qg}(x, y, z, t) \quad \quad \Lambda_{qg} = 1 \]

\[ \nu_{qg} = \left( \frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}| = \left( \frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 \left| \nabla_h \left[ \beta y + \nabla^2_h \psi + \frac{\partial}{\partial z} \left( \frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) \right] \right|. \]

\[ \nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left( \frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}|. \]
Movie: S. Bachman

Potential Temperature

Day 1

f-plane, spin-down

We’ll test this in a channel model, using three different resolutions:

The fastest growing mode is better resolved the higher the resolution, so the spindown will be slower for the coarser runs.

But the QG dissipation / diffusivity scheme is able to compensate!

Old Method=Smagorinsky viscosity with only implicit numerical diffusivity, no GM

New Method=QG Leith
Do the spectra behave?

Old Method = Smagorinsky viscosity with only implicit numerical diffusivity, no GM

Old Method in Red

New Method = QG Leith

Blue – dynamic QG Leith

\[ dx = \frac{L_d}{4} \]

\[ dx = \frac{L_d}{2} \]

\[ dx = L_d \]
But...we need to be careful of when QG isn’t appropriate:

- Stretching term can be too large when unstratified—use gridscale Burger number to determine when:

\[ Bu^* = \frac{N^*^2 \Delta z^2}{f^2 \Delta x^2} \sim Ro^*^2 Ri^* \]

\[ \frac{\nu_{qg}^*}{\nu_{2d}^*} \approx \frac{\nabla h q_{qg}^*}{\nabla h q_{2d}^*} \sim 1 + Bu^* \sim 1 + Ro^*^2 Ri^* \]

- Surface QG has different spectral characteristics—we have a theory, but simultaneous implementation unclear
Conclusions
Promising method: Realistic tests next!

- QG Leith=viscosity, Redi diffusivity, *and* GM transfer coeff.
  Nearly as suggested by Roberts & Marshall, 98, JPO
- Ensures $O(1)$ gridscale Reynolds & Péclet
- Revert to 2D Leith when QG is inappropriate
  - QG only if gridscale Burger near 1, gridscale Richardson>1

Our results suggest **QG Leith** will deliver the proven plug&play capability of LeithPlus with improved QG-based physics—Will matter most where stretching terms or APE balance are important, e.g., WBC.
Comparing the spectrum in QG Leith against another (inappropriate) LES closure, we see:

1) Better adherence to expected spectrum
2) Less “ski jump” near gridscale
3) Effects of choice *not limited* to small scales, slope in Smagorinsky run is too steep across whole range!
Fluxes:
Horizontal Buoyancy $\langle vb \rangle$

Parameterized:

Total:
Fluxes:
Vertical Buoyancy \(<wb>\)

Parameterized:

Total:
Fluxes:  
Momentum $\langle vw \rangle$  

Parameterized: 

Total: 