Global Stokes Drift and Climate Wave Modeling

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Conclusions

Hierarchy of Stokes Drift Approximations:

1. **2D spectral data known**: Use first-order 2D Stokes drift
   
   Random Error $\sim 10\%$

2. **1D spectral data known**: Use 1D wave spread approximation
   
   - 1D Unidirectional approximation is **not advised** since it systematically overestimates the 2D Stokes drift by approximately 33%

3. **Third-spectral-moment known**: Same as 1D wave spread at the surface
   
   Random Error $\sim 10\%$

4. **Third-spectral-moment unknown**: Use the second moment to empirically approximate the third moment

Climate Wave Model:

1. Unstructured node approach removes advective and directional singularities
2. Prototype model shows promise in great circle test case
**Introduction: Stokes Drift Velocity**

Stokes drift = mean( Lagrangian fluid velocity – Eularian current )

- Appears often in wave-averaged dynamics like Langmuir mixing
- Accuracy and data coverage remain challenges in global estimates
- Use of atmospheric data alone can be untrustworthy

*Figure:* 2D particle trajectories governed by the (a) linear and (b) nonlinear small-amplitude wave equations, and (c) the latter nonlinear mean drift over time (Kundu and Cohen, 2008)
Motivation: Importance for Climate Research

There is a persistent, shallow mixed layer bias in the Southern Ocean in global climate models (GCM): Langmuir mixing missing???

- Stokes drift plays a dominant role in determining the strength of Langmuir mixing
  \[ \frac{1}{La^2_t} \sim \frac{u^s(z = 0)}{u^*} \]

- Langmuir mixing is not currently in any GCM [2/2012]

Figure: Mixed layer depth bias is reduced in CCSM 3.5 model runs
Overview:

- Survey and error analysis of lower first-order Stokes drift approximations (spectral moments)
- Comparison of surface Stokes drift estimates using different data products (e.g., satellites, buoys, models) = Factor of 50% difference!
Global Wave Variable Examples

$U_{10}$ (cm/s): 2000/06/01

Surface Stokes Drift Magnitude (cm/s)

Magnitude Relative to Wind Speed (%)

$\cos(\text{WindDir} - \text{StokesDir})$
Stokes Drift and Wave Spectra Examples

The first-order Stokes drift magnitude depends both on the directional components of the wave field and the directional spread of wave energy.

\[
\mathbf{u}^S \approx \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^\pi \left( \cos \theta, \sin \theta, 0 \right) f^3 S_{f\theta}(f, \theta) e^{\frac{8\pi^2 f^2}{g} z} \, d\theta \, df \tag{1}
\]

Figure: Examples of wave spectra: (a) 2D spectra generated by WAVEWATCH III, (b) idealized directional spread (Holthuijsen), and (c) 1D Pierson and Moskowitz observational spectra (Stewart)
Stokes Drift and Multidirectional Waves

**Example:** Consider a bichromatic spectrum with the same amplitude and peak frequency for each monochromatic wave but separated by an angle of incidence $\theta'$.

\[
\mathbf{u}^S \approx \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^\pi \left( \cos \theta, \sin \theta, 0 \right) f^3 S_f(f, \theta) e^{\frac{8\pi^2 f^2 z}{g}} d\theta df
\]

Then the following relation holds\(^1\):

\[
\mathbf{u}_{bi}^S \neq 2 \mathbf{u}_{mono}^S = 2 \left( \cos \theta', 0, 0 \right) \mathbf{u}_{mono}^S
\]

Figure: Example of how the directional components of a wave field affect the magnitude of Stokes drift

\(^1\)The mean wave direction is along the $x$-axis
Stokes Drift: Improving 1D Estimates

Stokes drift error due to wave spreading in 1D approximates can be minimized by first recreating the 2D wave spectrum

Idea: Use an empirical directional distribution \( D_f \) to recover the 2D spectrum

\[
\int_{0}^{\infty} \int_{-\pi}^{\pi} S_{f\theta}(f, \theta) \, d\theta \, df = \int_{0}^{\infty} \left[ \int_{-\pi}^{\pi} D_f(f, \theta) \, d\theta \right] S_f(f) \, df = \int_{0}^{\infty} S_f(f) \, df
\]
Higher Order: Comparison of 2D and 1D Estimates

(a) Mag: 1D Unidirectional (x) vs 2D (y) (cm/s)

(b) Mag: 1D Spread (x) vs 2D (y) (cm/s)

(c) Mag: 1D Unidirectional (x) vs 1D Spread (y) (cm/s)

(d) Dir: Mean Wave (x) vs Stokes Drift (y) (rad)
Current State: Third-generation Wave Models

Current Model Basics:
- Uses structured grids (lat-lon, polar)
- Includes extensive physics and parameterizations

Current Model Deficiencies:
- Spatial and spectral singularities near the poles
- Performance declines as N/S boundaries are moved higher (presently $\pm 75^\circ$)
- Designed to forecast weather not climate

Figure: Spatial and spectral grid examples

Lat-lon grids:
- G3: $2.4 \times 3$
- G4: $3.2 \times 4$

Figure: WAVEWATCH III grid performance with benchmarking targets for coupling to NCAR CESM
**Unstructured Approach: RBF-Generated Finite Differences**

**RBF-Generated Finite Difference Method (RBF-FD):**

- Solves advective problems with near spectral accuracy
- Uses an unstructured node layout
- Allows geometric flexibility and local node refinement
- No advective and directional singularities
- Computational costs are spread equally
- Possibly well-suited for parallelization

**Great Circle Propagation Test Case:**

- 20 spatial $\times$ 10 spectral nodes
- Dissipation and dispersion error after 0.5 cycles
  - Third-order upwind $\sim$ 0.2
  - Radial Basis Functions $\sim$ $0.5 \times 10^{-4}$
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References

**Stokes Drift:**


**Langmuir Mixing:**


**RBF-Generated Finite Difference Method:**
