Bidirectional Reflectance Spectroscopy

2. Experiments and Observations

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Certain aspects of the theoretical bidirectional reflectance expression derived by Hapke [this issue] were studied experimentally. The theoretical expression satisfactorily describes the measured angular scattering and spectral properties of a cobalt glass powder whose absorptance and reflectance vary greatly with wavelength. The theoretical expression for the spectral single-scattering efficiency for irregular particles derived in paper I is shown to describe the measured efficiency of the cobalt glass particles. The predicted approximately linear dependence of the bidirectional espat-function \( W \) on the effective particle absorption optical thickness \( aD_e \) is shown experimentally to be valid for \( aD_e \) at least as high as 3. The theoretical reflectance function also describes measured distributions of intensity across the surfaces of planets, except near the sunlit limb, where the macroscopic roughness influences the brightness.

INTRODUCTION

In a companion paper [Hapke, this issue] (hereafter referred to as paper I) an analytic expression for the bidirectional reflectance of a semi-infinite particulate medium was derived (equation (16) of paper I). An expression for the scattering efficiency \( Q_s \) of an irregular particle was derived (equation (24) of paper I). An approximate relation between the single-scattering albedo \( \omega \), which can be found from the reflectance of a monomineralic substance in powder form, and the absorption coefficient \( \alpha \), which can be found from the transmission of a thin section, was found (equation (31) of paper I).

The study reported in this paper had the following objectives: (1) experimentally verify that the theoretical bidirectional reflectance function of paper I describes the angular scattering properties of a typical particulate surface whose absorptance and reflectance varies strongly with wavelength, (2) verify that the predicted linear relation between the espat-function \( W \), defined in paper I, and absorption coefficient \( \alpha \) exists and determine the range of \( D_e \), where \( D \) is the particle size, over which the linear approximation is valid, and (3) demonstrate that the theoretical limb-darkening curves describe observations on a variety of planetary objects.

LABORATORY MEASUREMENTS

Apparatus and Samples

The primary material chosen for the study was a glass made by adding 15% CoCl\(_2\) to a soda-silica (0.30 Na\(_2\)O, 0.70 SiO\(_2\)) glass. This material was chosen because it can be easily synthesized and processed and has an absorption coefficient which varies between 4 and 1500 cm\(^{-1}\) in the range of wavelengths easily accessible to ground-based planetary photometry. Thus by changing the wavelength it was possible to study the effects of varying the single-scattering albedo \( \omega \) over a wide range without affecting any other parameter. One portion of this glass was ground and polished to make a section 75 \( \mu \)m thick. The spectral absorption coefficient was measured by transmission methods using a Cary 14 spectrophotometer. The real part of the index of refraction was measured at 0.50 \( \mu \)m with oils and was used to correct for reflections from the front and rear surfaces of the thin section by the formula of Blain and Douglas [1965]. The resulting spectral absorption coefficient is shown in Figure 1. The remainder of the cobalt glass was pulverized and separated by wet sieving into various size ranges.

The angular scattering properties of surfaces made from the pulverized glass fractions were measured using a goniometric photometer. The photometer has a chopped quartz-iodine incandescent light source and a detector consisting of a photomultiplier, linear amplifier, and synchronous detector. The surfaces were measured at five wavelengths through interference filters having bandwidths of approximately 100 \( \AA \), centered at the wavelengths given in Table 1. The angular widths of the source and detector as seen from the surface were 0.5\(^{\circ}\). The detector position was held fixed at an angle of emission from the normal \( e = 30^{\circ} \), while the angle of incidence \( i \) was varied between 80\(^{\circ}\) on one side of the normal to 80\(^{\circ}\) on the other. The directions of the incident and emitted rays and the normal to the surface were coplanar. All measurements were relative to a BaSO\(_4\) diffusing standard illuminated at \( i = 0^{\circ}\) and viewed at \( e = 30^{\circ}\).

The spectral bidirectional reflectances of the powders were measured over the range 0.20-2.5 \( \mu \)m using a Cary 14 spectrophotometer with a reflectance attachment of our own design. The attachment consists of a box with two small plane mirrors, arranged so that one mirror reflects the incident beam.

Fig. 1. Spectral absorption coefficient of a soda-silica glass doped with cobalt; line: measured by transmission of a polished thin section; triangles: measured by reflectance of <37-\(\mu\)m powdered glass and using \( W = \alpha D_e \) with \( D_e = 36 \mu \)m.
TABLE 1. Scattering Parameters of Cobalt Glass Powders

<table>
<thead>
<tr>
<th>Filter*</th>
<th>Wave-length, Å</th>
<th>Powder, 37–74 μm</th>
<th>Powder, &lt;37 μm Packed</th>
<th>Powder, &lt;37 μm Sifted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w</td>
<td>b</td>
<td>B₀</td>
<td>h</td>
</tr>
<tr>
<td>U</td>
<td>3530</td>
<td>0.74</td>
<td>0.45</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>0.935</td>
<td>0.03</td>
<td>0.55</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.93</td>
<td>0.05</td>
<td>0.58</td>
<td>0.20</td>
</tr>
<tr>
<td>B</td>
<td>4480</td>
<td>0.58</td>
<td>0.05</td>
<td>0.35</td>
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<tr>
<td></td>
<td>0.89</td>
<td>−0.17</td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.82</td>
<td>0.10</td>
<td>0.58</td>
<td>0.20</td>
</tr>
<tr>
<td>G</td>
<td>5540</td>
<td>0.19</td>
<td>−0.10</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>0.52</td>
<td>−0.40</td>
<td>1.00</td>
<td>0.10</td>
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<td></td>
<td>0.37</td>
<td>−0.15</td>
<td>1.00</td>
<td>0.20</td>
</tr>
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<td>R</td>
<td>6900</td>
<td>0.26</td>
<td>−0.05</td>
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<td></td>
<td>0.68</td>
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<td>0.90</td>
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<td></td>
<td>0.54</td>
<td>−0.10</td>
<td>0.88</td>
<td>0.20</td>
</tr>
<tr>
<td>I</td>
<td>8200</td>
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<td>0.72</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.986</td>
<td>−0.13</td>
<td>0.65</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.973</td>
<td>0.04</td>
<td>0.32</td>
<td>0.20</td>
</tr>
</tbody>
</table>

*Bandwidths approximately 100 Å.

However, certain systematic effects observed in spectra of our samples led us to believe that the reflectance of our standard was somewhat different from the published values below about 0.25 μm and above about 1.7 μm. The far-UV reflectance of the standard was checked by measuring the spectral absorption coefficient of fused silica by transmittance and reflectance and adjusting the albedo of the standard until the results of the two sets of measurements agreed. The far-IR reflectance was obtained using a similar procedure with the cobalt glass. The resulting spectrum of the BaSO₄ standard is shown in Figure 2, along with the Grum and Luckey [1968] values. Neither of these adjustments is surprising. Practically every substance absorbs below 0.25 μm, so that any trace impurities could cause the spectrum of our standard to be lower than the published UV values. The region of 1.8–2.4 μm contains strong H₂O bands; thus slightly different degrees of hydration could account for the differences in the far-IR spectra.

Results

The angular scattering curves for the 37- to 74-μm and <37-μm size fractions of the cobalt glass are shown in Figures 3, 4, and 5. The <37-μm powder surfaces were prepared in two ways, sifting to form a loose, porous surface, and compacted. The scattering curve for the compacted powder is given in Figure 4, and the sifted in Figure 5.

Fig. 2. Absolute Lambert spectral reflectance of Eastman Kodak BaSO₄ paint. Dots: after Grum and Luckey [1968]; line: determined by comparison of transmission of thin section and reflectance of powder of pure fused SiO₂. See text for details.
The measured data were fitted by the theoretical function of paper I in the form of the radiance factor, or brightness relative to a Lambert surface illuminated normally

\[ r = \frac{w}{4} \frac{\mu_0}{\mu_0 + \mu} \left[ (1 + B(g, h))P(g) + H(\mu_0)H(\mu) - 1 \right] \]

where \( \mu_0 = \cos i, \mu = \cos e, g \) is phase angle, \( B(g, h) \) is the backscatter function defined in (13) of paper I, \( H(\mu) = \frac{(1 + 2\mu)/(1 + 2\mu(1 - \cos^2 g))^1/2}{1 + 2/(1 - \cos^2 g)} \), and \( P(g) \) is the single-particle phase function. A simple first-order Legendre polynomial expansion was used for the phase function: \( P(g) = 1 + b \cos g \). In this form the theoretical expression contains 4 adjustable parameters: \( w, b, B_0, \) and \( h \). The fitted theoretical curves are shown in Figures 3–5; the values of the adopted theoretical parameters are given in Table 1. The fit is seen to be excellent. The overall height and shape of the theoretical curve are controlled by \( w \). If the theory were grossly in error, no amount of adjustment of \( b, B_0, \) or \( h \) could bring it into agreement with the measurements, particularly for the high-albedo surfaces.

The parameters \( h \) and \( B_0 \) affect the reflectance only near zero phase, \( |g| \leq 20^\circ \). The width of the opposition effect near zero phase is controlled by the parameter \( h \) and the amplitude by \( B_0 \). As discussed in paper I, for powdered surfaces with a large dispersion in particle sizes with the particles in contact, \( h \) is of the order of 0.4 or less and depends on porosity but not on albedo. These predictions were supported by the measurements on the glasses. For the 37- to 74-\( \mu \)m and sifted <37-\( \mu \)m powders, \( h \approx 0.2 \), but \( h \approx 0.1 \) for the compacted <37-\( \mu \)m powder. The simple expression \( B_0 = \exp (-w/2) \) appears to roughly describe the amplitudes of the <37-\( \mu \)m powders. However, the amplitude function of the coarser 37- to 74-\( \mu \)m powder was smaller than this function over most of the range of \( w \). These results are in agreement with those of Hapke [1968], who found that coarser particles have a smaller opposition effect than finer ones. In fitting the opposition effect, the point at \( g = 1^\circ \) was given less weight than the points at larger phase angles because the brightness has such a large slope that the possibility of systematic error is high. In no case was it possible to fit the points between \(-2^\circ \leq |g| \leq -20^\circ \) by a backscatter function whose amplitude was not albedo dependent (see curve I of Figure 5). The data of Egan and Hilgeman [1976] also shows that the size of the opposition effect depends on albedo.

The sifted <37-\( \mu \)m surface has slightly lower effective values of \( w \) than the compacted <37-\( \mu \)m powder. This was attributed in paper I to effects of diffraction. It should also be noted that the particles tend to become more forward scattering (\( b \) decreases) as the albedo decreases, probably because of increasing relative contribution of surface Fresnel reflections.

In paper I a quantity called the effective single particle absorption thickness, or espat-function \( W \), was defined,

\[ W = (1 - w)/w \]

where \( w \) can be found from bidirectional reflectance measurements. For a monomineralic powder, \( W = (1 - Q_s)/Q_s \), where \( Q_s \) is the scattering efficiency; it was predicted that \( W \) would be linearly proportional to the absorption coefficient \( \alpha \) for \( W \approx 3 \). It was shown that in the linear range,
The spectral absorption coefficient was computed from $a(\lambda) = W(\lambda)/D_\alpha$, using $D_\alpha$ as an adjustable parameter independent of $\lambda$. In general, the agreement between $a$ found by transmission and by reflectance was satisfactory, except for large values of $W$, where departures were expected because the esp-function saturates. The values of $a(\lambda)$ measured by reflectance are shown in Figure 1 along with the values from transmission for the <37-µm powder. The two spectra are in agreement within ±15% over the entire wavelength range.

In Figure 6, $W(\lambda)$ calculated from reflectance is plotted against $a(\lambda)$ calculated from transmission. The effective diameter $D_\alpha$ is the slope of the line giving the best fit to the data at small values of $a$. It can be seen that $W$ is approximately linear with $a$ for $W \leq 3-6$. For the coarser particles with wider size distributions the upper limit of the linear range of $W$ is about 3, while for the 37- to 49-µm fraction, with the smallest size dispersion, $W$ is linear up to almost 6.

The cobalt glass has $n = 1.51$, so that from Figure 6 of paper I we find $S_e = 0.09$ and $S_i = 0.58$. Thus the theory predicts $D = D_\alpha/1.44$. For each distribution an average particle size $D$ was calculated using $D = D_\alpha/1.44$. These values of $D$ are given in Figure 6 and are acceptably in the middle of the sieved size ranges.

In paper I the following expression for $Q_s$ was derived:

$$Q_s = S_e + \left(1 - S_e\right) \left(1 - S_i\right) \left(r_i + \exp \left[-2(\alpha + S)D/3\right]\right) - \left(1 - S_e\right) S_i + \left(S_e - S_i\right) \exp \left[-2(\alpha + S)D/3\right]$$

where $r_i = (1 - [\alpha/(\alpha + s)]^{1/2})/(1 + [\alpha/(\alpha + s)]^{1/2})$ and $s$ is the internal scattering coefficient. For which a value of $s = 600$ cm$^{-1}$ (independent of $\lambda$) was necessary to describe the data at large optical particle thicknesses. Such a large value of $s$ is surprising; it is unlikely to correspond to intrinsic, distributed, internal scatterers because these would have interfered with the transmission measurements of the glass. The physical property corresponding to $s$ is unclear; one possibility is cracks or other imperfections near the grain surfaces induced by crushing.

From these results we conclude that the theoretical expression (24) of paper I for the single-particle scattering efficiency adequately describes the albedo of an irregular particle. We also conclude conservatively that the esp-function calculated from reflectance measurements can give accurate absorption coefficients if $W$ is less than about 3. If $W$ is greater than 3, the absorption coefficient will be underestimated relative to the smaller values, although it will still be within a factor of 2 of the correct value even for $\alpha D$ as large as 10. If the particle size distribution and refractive index of the powder are known, the spectral absorption coefficient may be estimated quantitatively. Conversely, from measurements of $a(\lambda)$, $n$, and $D$ of materials in the laboratory, the expressions of paper I may be used with confidence to calculate planetary spectra for comparison with observations.
PLANETARY LIMB-DARKENING OBSERVATIONS

Section 4l of paper I discusses how the bidirectional reflectance formula, equation (16), can be used to calculate the distribution of brightness across a planet, ignoring albedo variations. This equation should be accurate, except close to the limb where macroscopic roughness will affect the profile. To illustrate the limb-darkening, the theoretical expression was fitted to brightness profiles across several planets as measured by spacecraft. Mariner 10 profiles of the moon, Mercury, and Venus are shown in Figures 7, 8 and 9, respectively. The relative precision of these images are believed to be better than 3%, as discussed by Hapke et al. [1975] and Hapke [1976]. Figure 10 shows a profile of Mars as reduced by Thorpe [1973] from Mariner 9 images. The Mariner 9 vidicons suffered from a residual image problem (which was not present in the Mariner 10 cameras) so that the noise in the data is much larger on the Mars profile.

A single-parameter fit of the theoretical to observed profile was made by assuming that the particles are isotropic with single-scattering albedos which cause the predicted Bond albedos (equation (43) of paper I) to match the earth-based values. The theoretical profiles were then normalized to the center of the observed curves. Examination of Figures 7–10 shows that the agreement is quite reasonable. The only major discrepancies occur near the limbs of the low-albedo planets. As

Fig. 6. Generalized remission function \( W \) versus absorption coefficient for four size fractions of cobalt glass. \( W \) is calculated from the reflectance; \( \alpha \) is calculated from transmission; \( D_e \) is the slope of the data at small values of \( \alpha \) (dashed lines). Solid lines: theoretical values of \( W(\alpha) \) calculated from expressions of paper I using \( s = 600 \text{ cm}^{-1} \).

Fig. 7. Relative brightness profile along the luminance equator of the moon. Line: calculated from paper I with isotropic isotropic scatterers and \( \omega = 0.30 \); dots: Mariner 10 data. Phase angle is 83°. The abscissa is proportional to \( \sin \phi \).

Fig. 8. Relative brightness profile along the luminance equator of Mercury. Line: calculated from paper I with isotropic scatterers and \( \omega = 0.30 \); dots: Mariner 10 data. Phase angle is 77.5°. The abscissa is proportional to \( \sin \phi \).
discussed in paper I and by Hapke [1977], the limb discrepancy is caused by macroscopic surface roughness. It can be used to estimate average slopes by noting the angle by which the limb region must be tilted toward the spacecraft to reduce the limb brightness from predicted to observed. Mars and Mercury both apparently have mean slopes about half that of the moon, probably because of the weaker lunar gravity. The top of the cloud deck on Venus apparently has no appreciable roughness on a scale large compared with the photon mean free path in the clouds, consistent with earlier conclusions of Hapke [1976].

CONCLUSIONS

The theoretical expressions developed in paper I have been shown to be able to describe quantitatively the angular scattering properties of selected laboratory and planetary surfaces and to derive quantitative spectral absorption coefficients. Although this study was not exhaustive, the particulate surfaces investigated here are typical of those commonly encountered in laboratory and planetary work so that the theory should have widespread applications.

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REFERENCES


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